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THE ELEMENTS OF Arithmetick. IN THREE BOOKS.

Definitions.

1. UNity is that, by which every thing that is, is called one. *Unity is the beginning of every Number.*

2. Number is a Multitude compoſed of Units.

3. One Number is ſaid to meaſure another, when the leſſer being taken ſometimes is equal to the greater.

As 4 *meaſureth* 12, *becauſe thrice* 4 *makes* 12. *Unity meaſureth all Numbers.*

4. One Number is Multiplex of another, when the leſſer meaſureth the greater; or when the greater containeth the leſſer ſometimes preciſely.

5. The Aliquot part of a Number, is that which meaſureth the ſaid Number. The Aliquant part which does not meaſure it.

The Number 2, *is the Aliquot part of* 10, *becauſe it meaſureth* 10, *being taken* 5 *times; but* 3 *is the Aliquant part of* 10, *becauſe it does not meaſure* 10; *for being taken thrice it makes* 9, *and taken four times it makes* 12.

6. Like Aliquot parts are thoſe which meaſure their wholes equally; or which are equal times contained in their wholes.

10. 15. 2. 3. *2 and* 3 *are like Aliquot parts of the Numbers* 10 *and* 15; *becauſe both* 2 *in* 10, *and* 3 *in* 15 *are contained five times; or both* 2 *meaſureth his whole* 10, *and* 3 *its whole* 15, *by the ſame Number* 5.

7. Like Aliquant parts are thoſe which are contained in their wholes, as often as one another; and in what Remains are contained equal times like Aliquot parts of them.

Or, like Aliquant parts are thoſe which contain as many like Aliquot parts of their wholes as one another.

14. 28 8. 16 *8 and* 16 *are like Aliquant parts of the Numbers* 14 *and* 28; *becauſe that as* 8 *is contained once in* 14 *and* 6 *over, or thrice* 2, *that is* 3 *fourth parts of the* 8; *ſo* 16 *is contained once in* 28, *and* 12 *over, or thrice* 4, *which is alſo* 3 *fourth parts of* 16.

Or thus, 8 *and* 16 *are like Aliquant parts of the wholes* 14 *and* 28; *becauſe that as* 8 *containeth four ſeveral parts of the whole* 14, *to wit* 4 *times* 2, *ſo* 16 *containeth four ſeveral parts of the whole* 28, *to wit four times* 4.

8. In Numbers, the Ratio or Proportion is the mutual Habitude of two Numbers one to another, with reſpect to their exceſs, defect, or equality.

8. In

In every Proportion there are two Terms, of which that is called the Antecedent which is in the firſt Place, and the other the Conſequent; when the Antecedent is greater than the Conſequent, it is called a Proportion of greater inequality, or of the greater to the leſſer. When the Antecedent is leſs than the Conſequent, it is called a Proportion of leſſer in equality, or of the leſſer to the greater. When the Antecedent is equal to the Conſequent, it is called a Proportion of equality.

9. In Numbers, two Proportions are equal the ſame, or alike, (they all ſignifie one and the ſame thing) or four Numbers A, B, C, D. are ſaid to be Proportional when the leſſer terms of each Proportion are contained alike in the greater; and they are contained alike, if the leſſer (B, D,) are *a* ſimiler Aliquot parts of the greater A, C, *b* or like Aliquant parts.

A 8 B 2
C 16 D 4

a See def 6.
b See def. 7.

We expreſs equal Proportions thus; 8 *is to* 2, *as* 16 *is to* 4, *or* 8 *has the ſame reaſon to* 2, *as* 16 *hath to* 4, *that is* 8 .. 2 :: 16 .. 4.

10. When there are many equal Ratioes, and that the Conſequent of the firſt is the Antecedent of the next (ſo as that the mean terms are taken twice) the Proportion is ſaid to be continual, and the Numbers themſelves are call'd continual Proportionals. *We expreſs continual Proportions thus*; 1 is to 2, as 2 is to 4, and as 4 is to 8, and as 8 is to 16, *&c.* That is 1 .. 2 : : 2 .. 4 : : 4 .. 8 : : 8 .. 16.

11. But if the Conſequent of one of the equal Proportions is not the Antecedent of another, and ſo the mean terms are not taken twice, the Proportion will be diſcreet; and thoſe Numbers are ſaid to be

Proportional, without adding any other Word. *We expreſs diſcreet Proportions thus*; 9 is to 3, as 12 is to 4; that is, 9 .. 3 : : 12 .. 4.

12. When Numbers are continually Proportional, (A, B, C, D, E.) the reaſon of the firſt A to the third C, is ſaid to be duplicate of the reaſon which the firſt A, hath to the ſecond B; and the Ratio of the firſt A to the fourth D triplicate of the firſt A to the ſecond B.

A, B, C, D, E.
1 3 9 27 81
81 27 9 3 1

Of the Denominators and Compoſition of Ratios, ſee our Geometrical Elements, Book 5. Part 3.

13. A Number A, is ſaid to multiply a Number B, when the Number B, to be multiplied is ſo often taken, as there are Unites in the Number A multiplying.

Or thus; The Number A multiplieth the Number B, when a Number C is found, containing the multiplicand B ſo often as the Multiplyer A contains Units.

C 8 B 4
A 2

But Multiplication is more univerſally defined thus; The number A multiplieth the number B when a number C is found, which bears ſuch Proportion to the Multiplicand B, as the Multiplier A hath to a Unit.

The Number C *which is found, is called the Product or Fact; alſo when the Multiplyer* A *is a whole Number, the Product* C *is always greater than the Multiplicand* B, *as is manifeſt from Def.* 1. *and* 2. *but when the Multiplyer* A *is a Fraction, and leſs than a Unit, the Product* C *will be alſo leſs than the Multiplicand* B; *as is manifeſt from the laſt Definition.*

C $\frac{4}{3}$ B 4
A $\frac{1}{3}$ 1

Many numbers as 2, 4, 3. are ſaid to be Multiplied one by another; if 2 by 4 makes 8, and the Product 8 be multiplied into 3, for the laſt Product 24 is that

that which is made by the Multiplication of the numbers 2, 4, 3.

It is the same thing with Arithmeticians to draw a Number into a Number, as to multiply a Number by a Number; as A *by* B, *or* A *into* B *is all one.*

14. A number A is said to divide a number B, when a number C is found, shewing how often the Divisor A is contained in B.

And so the Divisor A *is a part of the Divided, or Dividend* B, *which is denominated from the Number found, (which therefore is called the Quotient.)*

B 12 C 3
A 4 1

Or more Universally. A number A divideth the number B, when another number C is found, having such Proportion to a Unite, as the Dividend B hath to the Divisor A.

15. One number A is said to measure another B, by any number X (which is called the Quotient) when A, the number measuring being taken so often as there are Unities in the Quotient X is equal to the number measured.

B 16 X 8
A 2 1

This Definition differs from the preceding. 1. Because Division may be performed, although the Divisor doth not measure the Dividend. 2. And also the Divisor be greater than the Dividend, as shall be shewn in its place; therefore Division appears to be of a larger extent then Mensuration.

16. An even number is that which may be divided into two equal parts: *Therefore some Number measureth every even Number by* 2.

17. An odd number is that which cannot be divided into two equal parts, or that which differeth from an even number by an unity.

18. A number evenly even, is that which an even number measureth by an even number.

Such is 24, *which the even Number* 6 *measureth by the even Number* 4.

19. A number evenly odd, is that which an even number measureth by an odd number.

Such is 12, *which the even Number* 4 *measures by the odd Number* 3,

20. A number odly odd, is that which an odd number measureth by an odd number.

Such is 21, *which an odd Number* 7 *measureth by an odd Number* 3.

21. A prime or incompoſit number is that which is meaſured only by a Unite, and conſequently hath no Aliquot parts beſides Unities.

Such are, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. *and infinite others. Alſo every prime Number is neceſſarily odd, for otherwiſe* 2 *would meaſure it.*

22. A compoſit number is that which ſome certain number beſides a unite meaſureth; *and conſequently hath ſeveral Aliquot parts beſides unites.*

Such are, 4, 6, 8, 9, 10, 12, 14, 16. *and infinite others.*

23. Numbers prime the one to the other, are ſuch as no other common meaſure meaſureth but a unite.

The numbers 15 *and* 38 *are ſaid to be prime the one to the other, becauſe that although ſome numbers meaſure each, and ſo neither is a prime number, yet no number meaſureth both; in like manner,* 8, 10, 15. *are three numbers Prime to one another, becauſe no number meaſureth them all three, and ſo on.*

24. Numbers compoſit the one to the other, are they which ſome number (beſides a Unite) being a common meaſure, doth meaſure.

One of the given numbers may be the common meaſure.

The

The two numbers 8 and 10, are compoſit the one to the other, becauſe 2 meaſureth both, likewiſe 5 and 15. are compoſit to one another. becauſe 5 meaſureth both its ſelf and 15; the three numbers 3, 9, 12. are compoſit to one another, becauſe 3 meaſureth 9 and 12. and its ſelf; 6, 10, 12, 16. are four numbers compoſit to one another, becauſe 2 meaſureth all four.

25. A plain number is that, which is produced from the multiplycation of two numbers; And the numbers which multiply one another, are call'd the ſides of the ſaid plain number.

Therefore every plain number is compoſit.

So 12 is a plain number, becauſe it is made by the multiplycation if 6 by 2, alſo 24 is a plain number, becauſe it is made of 4 by 6.

26. A Solid number is that which is produced from the multiplycation of three numbers, and the numbers that multiply one another are called the ſides of the ſaid ſolid number.

Therefore every ſolid number is compoſit.

24 is a ſolid number becauſe it is made by the multiplycation of three numbers 2, 3, 4. for 2 x 6 = 6 and 6 x 4 = 24.

27. Like plain and ſolid numbers are they, whoſe ſides are proportional.

6, 24, are like plains; whoſe ſides are 2, 3 and 4, 6, for 2. . 3 :: 4 . . 6.	2, 3. 6. 4, 6.
alſo 24. 92. are like ſolid., becauſe the ſides of one, 2, 4, 3, are proportionable to the ſides of the other, 4, 8, 6.	2, 4, 3. 24. 92. 4, 8, 6.

28. A ſquare number is that which is made by the multiplycation of two equal numbers, or by the multiplycation of any number by its ſelf, which is called a ſquare root.

The firſt ſquare is 4, which is made of 2 by 2 or of 2 multiplyed into its ſelf; the ſecond is 9, which is made of 3 in its ſelf, and ſo on in infinitum.

29. A Cube is that which is made by the multiplycation of three equal Numbers or of the ſame number thrice taken, which is called a Cube root. *The*

The first Cube is 8 *which is made by the Multiplication of* 2 *thrice taken,* 2, 2, 2, *for* 2 x 2 = 4 *and* 4 x 2 = 8 *the second is* 27 *which is made by the multiplication of* 3 *thrice taken,* 3, 3, 3; *for* 3 x 3 = 9 *and* 9 x 3 = 27.

30. A perfect Number is that which is equal to all its aliquot parts.

The first perfect number is 8 *for all the aliquot parts thereof are* 1, 2, 3, *which together make* 6. *The Second* 28, *for all its aliquot parts are,* 1, 2, 4, 7, 14. *which together make* 28, *of these see the last Prop. Book* 9. *and its Scholium.*

AXIOMS.

A. B. Z.

1. Numbers, A, B, which are æquimultiplices of the same number Z are equal, and numbers which are æquimultiplices of equal Numbers are equal.

2. Numbers A, B, whose Equimulplex is the same Number are equal, and those Numbers are equal whose equimultiplices are equal.

3. Those Numbers are equal which are the same part of the same Number, as a half, a third, or a fourth, *&c.*

And those Numbers are equal, which are the same part of equal Numbers.

4. Those Numbers are equal, of which one Number is the same part, and those Numbers are equal of which equal Numbers are the same part.

5. Unity measures every Number by the Units that are in it, that is by the same Number.

6. Every Number measureth its self by a Unite.

A 4. B 3. C. 12.

7. If one Number A, multiplying another B generateth any Number C, the multiplyed B measureth the generated C, by the multiplyer A.

This is manifest from the 13. *and* 15 *Definit.*

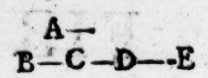
8. If a Number A measureth or divideth another number C. by the quotient B, also the quotient B multiplying

tiplying the measuring A, produces the measured C, or the measuring A multiplyed by the quotient B, shall produce the measured C.

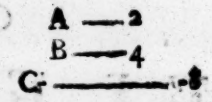
This is manifest from Defin. 13.15.14.

9. That a greater than any Number may be taken.

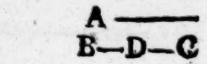
10. If a Number A measures how many Numbers soever BC, CD, DE, it also measureth, the Number composed of them BE.



11. If a Number A measureth any Number B, it also measureth every Number C, which the said Number measureth.



12. The Number A that measures the whole BC, and the part taken away BD doth also measure the residue DC.



THE

THE FIRST BOOK OF THE ELEMENTS OF Arithmetick,

And the Seventh of EUCLID.

In the Citations of the Books, Euclid's *number is retained.*

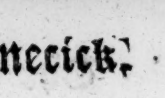
PROP. I.

IF from two unequal Numbers the leſſer be continually taken from the greater by an alternate Subtraction; and the remainer do never meaſure the preceeding, until unity be come to: then are the given numbers (a) prime the one to the other.

(a) See def. 14.

Let

Let the two Numbers, A B and C D be given; now C D being taken from A B so often as it can leaveth E B, and E B being taken so often as it can from C D, leaveth F D, and F D being taken so often as it can from B E, leaveth a Unity. I say that the Numbers A B, C D, are Prime the one to the other.



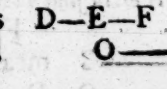
If it can be, let the Numbers A B, C D, have a common measure H, therefore because you will have H, to measure C D, and C D measures (*b*) A E, also (*c*) H shall measure A E; but you will have H also to measure the whole A B, therefore also (*d*) it measureth the remainer E B, but E B measureth (*e*) C F, therefore also H (*f*) measureth C F; wherefore seeing you would have H, to measure the whole C D, it will also measure the remainer (*g*) F D; But F D measureth (*h*) E G, therefore also H measureth (*i*) E G; but I have shewn above that H also measureth the whole E B, therefore also it measureth the remainer (*k*) G B, a number a unite which is absurd.

(*b*) *Hyp.*
(*c*) *Ax.* 11.
(*d*) *Ax.* 12.
(*e*) *Hyp.*
(*f*) *Ax.* 11.
(*g*) *Ax.* 12.
(*h*) *Hyp.*
(*i*) *Ax.* 11.
(*k*) *Ax.* 12.

PROP. II.

TWO Numbers (A B, D F) given, not Prime the one to the other, to find their greatest common Measure.

The lesser D F, taken from the greater A B so often as it can leaveth C B, C B taken so often as it can from D F leaveth E F and so on.



By this alternate Subtraction at length is left some number which measureth the foregoing, for if we came to Unity by Subtraction, the given numbers (*a*) would be Prime contrary to the Hyp: Therefore let the remainer be E F, and let it measure the preceeding

(*a*) *Preced.*

ing C B, I say that E F is the greatest common measure of the numbers A B, and D F, which shall be thus demonstrated.

(b) Constr. (c) Constr. (d) Ax. 11. (e) Ax. 12. (f) Constr. (g) Ax. 11. (h) Constr. (i) Ax. 10.

E F measureth (b) C B, and C B measureth (c) D E, therefore also E F measureth (d) D E, but E F also measureth its self, therefore E F (e) the whole D F; but D F, measureth (f) A C, therefore also E F, measureth (g) A C, but E F also measureth (h) C B, therefore also E F measureth the whole (i) A B, therefore E F is the common measure of A B, and D F.

(k) Ax. 11. (l) Ax. 11. (m) Constr. (n) Ax. 11. (o) Ax. 12.

And that it is the greatest is thus shewn; If it can be let there be another O, greater than E F, because you will have O to measure D F; and that D F measures A C, also O, shall measure (k) A C; But you will have O also to measure A B, therefore also O shall measure (l) C B, but C B (m) measureth D E, therefore also O measureth (n) D E, wherefore seeing you would have O to measure the whole D F, it also measureth the (o) remainer E F, lesser then its self which is absurd.

Corollary.

A Number (O) that measureth two Numbers (A B, D F) also measureth their greatest common measure (E F)

PROP. III.

THree Numbers given (C, D, E) not Prime to one another to find their greatest common measure.

(a) Prec.

C ——— B
D ——— O
E ——— P
S ———

Find O the greatest (a) common measure of the two Numbers given C, D, If O also measures E; it will be the greatest common measure of the three Numbers C, D, E; for if another S were greater then O; then also S, would measure O by the precedent Coroll. which is absurd, seeing S. is put greater than O. But if O does

does not measure E, yet O and E will be compositt the one to another:

For seeing C, D, E (*b*) are three numbers composed to one another, some common measure will (*c*) measure them, and also the said (*d*) O: therefore I say that P. the greatest common measure found of the two numbers O, E is the greatest common measure of the three Numbers C, D, E.

(*b*) *Hyp.* (*c*) *De.* 24. (*d*) *Cor. prec*

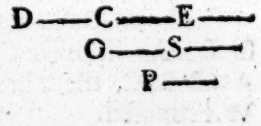
For seeing P (*e*) measureth O, and O measureth (*f*) C, D, also P (*g*) measureth C, D; but (*h*) P. also measureth E, therefore P. measures the three C, D, E; and that it is the greatest common measure 'tis thus shewn; let another S. if it can, be greater than P, therefore because S. measureth C, D, E, it will also measure (*i*) G the greatest common measure of C, D: Wherefore because S measureth E and O; S will again (*k*) measure their greatest common measure (*l*) P, the greater the lesser, which is absurd.

(*e*) *Constr.* (*f*) *Constr.* (*g*) *Ax.* 11. (*h*) *Constr.* (*i*) *Corol.* (*k*) *Corol.* (*l*) *Constr.*

Corollarys.

1. After the same manner may be found the greatest common measure of four, or more Numbers not Prime the one to the other.

2. The Number S. measuring any Numbers E, D, C, also measureth their greatest common measure P, which is manifest from the last part of the preceding Demonstration.



PROP. IV.

THE Fourth and rest of the Propositions to the 15. *inclusive are contained in the Universal Propositions of the Fifth Book of our Elements of Geometry.*

PROP.

PROP. XVI.

IF two Numbers (A, B) multiplying themſelves mutually the one by the other, produce the Numbers C, D, the Numbers produced are equal to one another.

Becauſe A multiplying B maketh D, it will be (*a*) 1 .. A :: B .. D therefore by permutation (*b*) 1 .. B :: A .. D, again, becauſe B multiplying A, makes C, it will be (*c*) 1 .. B :: A .. C, therefore A .. D :: A .. C. therefore alſo (*d*) C = D. W. W. Dem.

(*a*) Def. 13. 7.
(*b*) 6. 5.
(*c*) Def. 14. 7.

Unity 1
A 6 B 4
C 24 D 24

Corollary.

A. B. C.

1. IF a Number A, multiplying another number C, produces a third C. the multiplyer A, will meaſure the Product C. by the Multiplicand B. for becauſe A x B = C, alſo B x A = C by this Prop. wherefore ſeeing A which at firſt was the multiplyer is now the multiplycand, in reſpect of the ſame Product C, it is manifeſt from Ax. 7. that A meaſureth C by B.

2. If A meaſureth or divideth C by B; alſo the quotient B ſhall meaſure the ſame C. by A.

For becauſe A meaſureth C. by B, therefore by Ax. 8. B x A will make C, wherefore by this 16 Prop. alſo A x B = C, therefore by Ax. 7. B meaſures C by A. W. W. Propoſed.

PROP. 17.

IF a number (A) multiplying, Numbers (B, C,) how many ſoever, produce ſo many other Numbers AB, AC. the Numbers produced of them ſhall be proportional to the Numbers multiplyed.

COL-

For ſeeing A x B = A B, it will be (a) (a) *def.*
Unity 1 . . A : : B . . A B, again ſeeing A x C = . . 7.
1 A C it will be (b) 1 . . A : : C . . A C (b) *ibid.*
A therefore (c) B . . A B : : C . . A C (c) 11. 5.
B. C. therefore by permutation B . . C : : (a) (d) 16, 5.
A B. A C. A B . . A C. W. W. Demonſtrated.

SCHOLIUM.

We have here begun to expreſs the product of Multiplycation, only by the mutual appoſition of the Multiplying Numbers; of which ſee what has been wrote before the beginning of this Seventh Book: if this method is uſed (which for the moſt part will be wonderful compendious) the 17. Propoſion may be more commodiouſly expreſt after this manner.

Numbers how many ſoever A B, B C having one common ſide B have the ſame proportion the one to the other as the other ſides A and C have.

The ſides of a number are they; by the Multiplycation whereof the ſaid number is produced; moreover that tyroes may the better uſe themſelves to this method: Which (as I ſaid before) is very commodious and compendious, beſides the Example brought in this Propoſition; I think meet to add ſome other Examples.

1. A A . . A B : : A . . B
2. A B . . B B : : A . . B
3. A A A . . A A B : : A . . B
4. A B B . . B B B : : A . . B

In the 1 *the common ſide is* A. *and there are left the ſecond* A, *and* B; *In the* 2. *the common ſide is* B, *and there remains* A *and* B, *In the* 4, *the common ſide is* BB *and the remains are* A *and* B.

5. A, AB, BB. *are in continual proportional in the reaſon of* A *to* B *which is manifeſt from* 1 *and* 2.

6.

6. A A, AAB, ABB, BBB, *and in continual proportionals in the reason of* A *to* B, *as is manifest from* 3 *and* 4.

9. AB, AC, AD, AE, AF, *have the same proportion the one to the other, as* B, C, D, E, F *have, which is manifest from the Proposition.*

What great use there is of this Scholium, for the easie expediting of prolix and difficult Demonstrations, will appear both in these Books, and in Practical Arithmetick.

PROP. VIII.

IF Numbers B, C, *how many soever multiplying the same Number* A, *produce so many other Numbers* D, F, *the numbers produced* D, F *are proportional to the Multiplying numbers* B, C.

B 2. C 4.
A 3.
D 6. F 12.

For seeing B x A = D also (*a*) A x B = D; and seeing C x A = F also (*b*) A x C = F, therefore (*c*) B . . C : : D . . F, W. W. Dem.

(*a*) 15. 7. (*b*) *ibid.* (*c*) *Praeced.*

Corollary.

A Number A dividing or measuring any numbers, J. P. produceth Quotients (R, S) proportional to the numbers divided or measured.

For because A, measureth or divideth the said J. P. by the Quotients R. S it is manifest by the 8*th.* axiom that A x R = I and A x S = P. therefore by the 17 Prop. I . . P : : R . . S.

Other ways.

(*d*) *def* 14. I 12. P 26.
(*e*) 16. 5. A 4
(*f*) *def.* 14. R 3. S 5.
Unit E

WHen A measuring or dividing A maketh R it will be (*d*) 1 . . R : A . . I therefore (*e*) by permutation 1 . . A : : R . . I. again seeing A measuring P makes S, it will be (*f*) 1 . . S : : A . . P. therefore by permutation 1 . . A : : S . . P wherefore seeing I have already

ready ſhewn that alſo R . . I : : I . . A. it will be (g) R . . I . : S . . P. and by permutation R . . S : : I . . P. which was to be Demonſtrated. (g) 11. 5.

PROP. XIX.

IF four Numbers A, B, C, D, *are proportional; the number* A D *produced of the firſt* A. *and fourth* D *will be equal to the number produced of the ſecond* B *and third* C. *and the contrary.* (a) 18. *or* 17. 7.

1 Part. A x C = AC. AC . . AD : : (a) C . . D and AC . . BC : : (b) A . . B wherefore ſeeing (c) A . . B : : C . . D alſo (d) AC . . AD : : AC . . BC therefore (e) AD = BC W. W. Demon.

A 3. B 2. C 6. D 4. EC 12. AD 12. A C.

(b) *by the ſame.* (c) *Hyp.* (d) 11. 5. (e) 9. 5.

2 Part becauſe by hypotheſis AD = BC, it will be (f) AC . . BC : : AC . . BC but AC . . BC (g) A . . B and AC . . AD : : C . . D therefore (h) A . . B : : C . . D. W. W. Demonſt.

(f) 7. 5. (g) 18. 7. (h) 11. 5.

Corollary 1.

IF two numbers B, C, meaſure or divide the ſame Number A, by the quotients O, P. it will be B . . C : : P . . O.

For becauſe B. C, meaſures or divides A. by O, and P. therefore by ax. 8. B x O = A and P x C = A that is B x O = P x C. Therefore by this Propoſition B . . C : : P . . O.

A
B. C.
O. P.

Corollary 2.

IF the ratio A . . B be greater than C . . D; then A D produced of A x D ſhall be greater than B C produced of B x C. and the contrary.

Part 1 A x C = AC, AC . . BC : : (a) A . . B. that is greater (b) than C . . D that is again (c) greater then AC . . AD, therefore BC is leſſer (d) then AD.

A 4. B 2. C 6. D 4. AD 16. BC 10. A C

(a) *Sch. pro.* 17. 7. (b) *Hyp.* (c) *Sch. pro.* 17. (d) 19. 5.

Part 2. Becauſe AD is already agreed to be greater then BC the ratio AC . . BC ſhall be greater then the ratio (*e*) AC . . AD, but AC . . BC :: (*f*) A . . B and AC . . AD :: (*g*) C . . D, therefore alſo the ratio A . . B is greater then that of C . . D.

(*e*) 8. 5.
(*f*) *Scho.*
(*g*) *the ſame.*

PROP. XX.

IF three numbers (A B C) are proportional, the number (A C) produced by the extreams is equal to the ſquare (B B) of the mean.

And if the number contained under the extreams be equal to the ſquare of the mean, the three numbers are proportional.

Part 1. Write the mean B twice, therefore by *hyp.* A . . B :: B . . C ſeeing now four proportionals are had, the number *a* generated from the extreams AC, will be equal to the number generated from the ſecond B and third B, that is to the ſquare of the ſaid B.

A 4. B 8.
B 8. C 16.
BB 64. AC 64.

a 1 *Part præced.*

Part 2. Is Demonſtrated after the ſame manner, from Part the ſecond of the foregoiug.

PROP. XXI.

NUmbers (A and B) being the leaſt of all in their proportion do equally meaſure the Numbers C, D, having the ſame proportion with them.

For ſeeing by *hyp.* A . . B :: C . . D, alſo by permutation A . . C :: B . . D, therefore A and B are *a* like aliquot or aliquant parts of C and D, (and that A B are leſs then C, D, is manifeſt from the *hyp.*) but that they are not like aliquant parts I thus ſhew if A, B are like aliquant parts of the ſaid C, D *b* then A containeth O ſuch an aliquot part of the ſaid C, as B containeth of the ſaid D, ſuppoſe P. and conſequently *c* O . . C :: P . . D and by per-

A 2. B 3.
O P
C 6. D 9.

a def. 9.
b def. 7.
c def. 6.

permutation *d* O . . P : : C . . D that is A . . B but A, B, contain the ſaid O, P. therefore A, B are not the leaſt of all that *have* the ſame proportion, which is againſt the *hyp.* Therefore A B are not like aliquant parts of the ſaid C, *D.* it remains therefore that they are like aliquot parts, and conſequently do equally meaſure the ſaid C, D. W. W Demon. *d* 16. 5.

Corollary.

AFter the ſame manner it will be Demonſtrated that Numbers A, B, C, D, E, how many ſoever being the leaſt of all that have the ſame proportion with them do equally meaſure ſo many numbers in the ſame proportion with them. O, P, Q, R, S, F, G, H, I, K.

PROP. XXII.

This is contained in Prop. 22. Book 5.

PROP. XXIII.

NUmbers a prime the one to the other A, B, are the leaſt of all that have the ſame proportion wit o them. A 15. B 8. C D E Unity. *a def.* 23.

If it be poſſible, let others C, D: be the leaſt and proportional to the ſaid A, B therefore C, D, meaſure *b* A, B equally that is by the ſame number E ; Wherefore becauſe C meaſures A by E; 1 . . E : : C . . A, and by permutation 1 . . C : : E . . A, but Unity meaſures C. therefore alſo E meaſures A, after the ſame manner it may be ſhewn that E meaſures B. ſeeing therefore that E meaſures both A and B, they are not prime the one to the other contrary to the hypotheſis. *b* 21. 7. *def.* 23.

Corollary.

A, B, C, D,
F, G, H, I.
E
Unity.

AFTER the ſame manner it may be demonſtrated that if Numbers how many ſoever A, B, C, D, E, are Prime one to the other, they are the leaſt of all Numbers, having the ſame proportion with them.

PROP. XXIV.

NUmbers A, B, being the leaſt of all that have the ſame proportion with them, are Prime the one to the other.

A, 4. B, 5.
E.
C D

If they are not Prime, let the common meaſure E, meaſure A, by C, and B, by *A*, whence *a* A.. B:: C.. D, wherefore ſeeing C, D, are leſs then A, B, A, B are not the leaſt of all in the ſame proportion with them, which is contrary to the Hypotheſis.

a Cor. pr. 18. 7.

Corollary.

A, B, C, D,
E,
F, G, H, I,

AFter the ſame manner it may be demonſtrated, that if Numbers how many ſoever A B, C, D, are the leaſt in their proportion they are alſo Prime the one to the other.

PROP. XXV.

IF a Number (N) meaſures one (A) of two Numbers (A, P) being Prime the one to the other, it ſhall be Prime to the other Number (B.)

For

For if N, B, are not Prime the one to the other, let X. measure both, wherefore because X. measureth N, and N measureth *a* A, therefore also X, will measure *b* A, but you would have X, to measure B, therefore A B, are not Prime the one to the other, contrary to the Hypothesis.

A,	B,	
N,	X,	*a Hyp.*
		b ax. 11.

PROP. XXVI.

IF two Numbers (A, B,) are Prime to any Number (C,) the Number (A B) produced of them shall be Prime to the same (C)

For if A B and C, are not Prime the one to the other, let D measure both by F, therefore D x F, *a* = A B but also A x B *b* = A B, therefore D . . A : : *c* B . . F, now because A and C. are Prime the one to the other, and that you would have D to measure C, therefore *d* D will be Prime to A, wherefore D and A, are the three *e* least in their proportion, and therefore they equally measure *f* B. F. in the same proportion with them, to wit, D measures B; and A the said F, and because you would have D, also, to measure C, therefore D shall measure both C, and B, and therefore C, and B, are not Prime the one to the other, contrary to the Hypothesis, therefore AB, shall be Prime to C. W. W. demonstrated.

A. 7.	B. 3.	
C 8.		*a ax.* 8.
A. B. 21.		*b Hyp.*
D — F —		*c* 19. 7.
		d prac.
		e 23. 7.
		f 21. 7.

PROP. XXVII.

IF two Numbers (A, B,) are Prime the one to the other, also the Square (A A) of one of them shall be Prime to the other (B.)

Write A twice. now because the first A *a* is Prime to B also the last A, shall be Prime to B; therefore the fact of A x A, that is, *b* A A. is Prime to B.

A 4.	B 7.	
A A 16.		*a Hyp.*
A.		*b prac.*

PROP. XXVIII.

IF two numbers (A B) *are prime to two numbers* (C D) *each to either of both; the numbers alſo produced of them* (A B, C D) *ſhall be prime to one another.*

(a) *Hyp.*
(b) 26. 7.
(c) *Hyp.*
(d) 26. 7.

A. B.
AB.
C. D.
CD.

For becauſe (a) A and B are prime to C, alſo (b) A B ſhall be prime to C: again becauſe A and B are prime to (c) D, alſo AB ſhall be prime to D; ſeeing therefore C and D are prime to AB, alſo CD, the Number produced of them, ſhall be (d) prime to AB. W. W. Demonſt.

PROP. XXIX.

IF two Numbers (A, B) *are prime the one to the other, their Squares* AA, BB, *and Cubes*, AAA, BBB, *and ſo on, ſhall alſo be prime the one to the other.*

(a) 27. 7.
(b) 27. 7.

A B
AA BB
AAA BBB
AAAA BBBB

Becauſe A B are prime the one to the other; alſo AA is (a) Prime to B. and becauſe AA, and B are prime the one to the other; alſo (b) BB ſhall be prime to AA, which was to be firſt Demonſtrated.

(c) 27. 7.
(d) 28. 7.

Again becauſe A, B are prime to one another, alſo BB will be prime to (c) A; therefore B and BB are prime to A: It is alſo already ſhewn, that A and AA are prime to B; wherefore AAA, produced of A x AA is prime to (d) BBB, produced of B x BB, which was the other.

(e) 26. 7.
(f) *ibid.*

Alſo becauſe A and AA are prime to B, alſo AAA produced of them is prime to (e) B; after the ſame manner, becauſe B x BB are prime to A, alſo BBB ſhall be prime to (f) A; now becauſe A and AAA is prime to B, and B and BBB alſo prime to A, the pro-

products AAAA, and BBBB, produced of them shall also be (g) prime to one another; and so on infinitely. (g) 28. 7.

PROP. XXX.

IF two Numbers A, B, are prime the one to the other; also both together A † B shall be prime to either of them.

And if two numbers together (A † B) be prime to any one of them; also the numbers given shall be Prime the one to the other.

Parſ. 1. For if A † B, is not prime to A, let the Number C measure them, which also must (a) measure B; and then A, B, are not prime the one to the other, which is against the Hypotheſis. (a) Ax. 11.

A † B
C ——

Parſ. 2. If A, B, are not prime to one another, let C measure them, which consequently (b) will measure A † B, contrary to the Hypotheſis. (b) Ax. 12.

Corollary.

IF A † B be prime to A, also A † B shall be prime to B; for because A † B is prime to A, A and B shall be prime the one to the other, by Part 1. then because A and B are prime to one another, also by part. 1. A † B shall be prime to B.

PROP. XXXI.

EVery prime Number (A) is prime to every other Number B, that it does not measure.

For if A and B are not prime to one another, some Number C different from A, will measure them; but A, by Hypothes doth not measure B; therefore (a) C is not a prime Number. (a) Def. 21

A. B. C.

PROP. XXXII.

(a)Def.25 *IF a prime Number C, measures a plain* (a) *Number A B, it shall also measure one of the sides of the plain Number.*

Let the prime Number C measure the plain Number A B by D; if now C does not measure A, then C, A (*b*) shall be Prime to one another, and therefore the (*c*) least in the their Proportion; but because C measureth AB by D, therefore C x D = (*d*) AB; but also A x B = AB, wherefore C.. A :: (*e*) B.. D. Wherefore seeing *o* C, A are the least in the same Proportion, C will measure (*f*) B; after the same manner it will be proved that C measures A, if it does not measure B. Therefore the Proposed is manifest.

(*b*) *præc.* (*c*) 23. 7. (*d*) *Ax.* 8. (*e*) 19. 7. (*o*) 23 7. (*f*) 21 7.

A, B, AB, C. D—

PROP. XXXIII.

SOme prime Number measureth every Compositt Number.

Let any compesit Number F be given; now one or more Numbers, whereof let the least be N (*a*) measures it. I say that N is a prime, for if it is not, some other P will measure it, which then will also measure (*b*) F, although it be less then N, which is absurd; seeing N is put for the least of all those that measure F.

(*a*) *def.* 22. (*b*) *Ax.* 11.

F, N—P—

PROP. XXXIV.

EVery Number is either a prime, or may be measured by some prime.

This is manifest from the preceeding Prop.

PROP.

PROP. XXXV.

NUmbers (A, B, C) howmany ſoever being given, to find the leaſt in the ſame Proportion with them.

Find the greateſt common meaſure (*a*) O of the given Numbers A, B, C, which dividing or meaſuring them will quote or make D, E, F; I ſay that theſe are the leaſt in the ſame Proportion with the given Number A, B, C; if they want a common meaſure, they are prime the one to the other, by Th. 7. and ſo the leaſt in their Proportion by 23. 7: But that D, E, F, are Proportional is manifeſt from the Corall. to Prop. 18. 7; and that they are the leaſt I thus ſhew; If it be poſſible let others P, Q, R, be the leaſt in the ſame Proportion with A, B, C, given, therefore P, Q, R, meaſure (*b*) A, B, C by the ſame Number, as by X. Therefore P x X = (*c*) A, and alſo D x O = (*d*) A, wherefore (*e*) P.. D:: O.. X; but you would have P to be leſs than D, therefore alſo O is leſs then X, now becauſe P, Q, R meaſure A, B, C by X, it is manifeſt that P, Q, R multiplyed by X (*f*) produces A, B, C; and conſequently that X meaſures (*g*) A, B, C by P, Q, R; Therefore O is not the greateſt common meaſure of the ſaid A, B, C given, which is againſt the Hypotheſis.

A.	B.	C.
	O	-—
D,	E,	F
P,	Q,	R
	X	-—

(*a*) 3. 7.
(*b*) 21. *cor.* 7.
(*c*) *Ax.* 8.
(*d*) *Conſtr.* & *Ax.* 8.
(*e*) 19. 7.
(*f*) *Ax.* 8.
(*g*) *Ax.* 7.

Corollarys.

THE greateſt common Meaſure of Numbers how many ſoever, meaſureth them by the leaſt of all the numbers, which are in the ſame proportion with them.

PROP.

PROP. XXXVI.

TWO Numbers (A,B,) being given to find the leaſt number that they do meaſure.

A. B. AB O N. P.

If the given numbers A,B are prime to one another, the number AB produced of them, is that ſought; for becauſe A x B = A B, A ſhall meaſure *a* A B, and becauſe B x A = *b* A B, B ſhall meaſure AB, therefore both A and B meaſure AB, but that A B is the leaſt number that A and B meaſure, I thus ſhew; If it be poſſible let O be leſs, and let A and B meaſure the ſaid O by N and P, wherefore *c* A .. B :: P .. N, but becauſe A B, are *d* prime numbers, they are the *e* leaſt in their proportion, and conſequently A meaſures P, and *f* B meaſures N; Now becauſe A x B = A B and A x N = O as is ſhewn above, it will be *g* B .. N :: A B .. O, but I have already ſhewn that B meaſures N, therefore alſo A B will meaſure O, leſſer than its ſelf which is abſurd.

a ax. 7. *b* 16. 7. *c* cor. 19. 7. *d* Hyp. *e* 23. 7. *f* 21. 7. *g* 17. 7.

A. B C. D. A D or B C O N. P.

If the given Numbers A,B are not prime to one another, find C. D the *h* leaſt in the ſame proportion with them, then A x D = A D or B x C = B C I ſay that A D or B C (for they are *i* equal) is the leaſt which A, B meaſure; for if you would have A and B to meaſure O, ſome other leſs than A *D*. I will ſhew by the like reaſoning as above that A D will then meaſure O, leſs than its ſelf.

h præc. *i* 19. 7.

PROP. XXXVII.

IF two Numbers (A *and* B) *meaſure any other Number* (C O) *alſo* (F) *the leaſt, which the ſaid* (A,B,) *meaſure, ſhall meaſure the ſame* (C O.)

For

For if F doth not measure C, O, it being taken from C O, so often as it can, leaves D, O, less then its self, but *a* because both A and B measure F, and that F measures C D, also *b* A and B shall measure the said C D; but A, B do also *c* measure the whole C O, therefore also they shall measure the remainer *d* D O less then F, which is against the Hypothesis.

A. B.
C——D——O
F

a Hyp.
b ax. 11.
c Hyp.
d ax. 12.

PROP. XXXVIII.

THREE Numbers (A, B, C,) being given to find the least number which they measure.

Find the *a* least P which A and B, measure, if the third C, doth also measure P, then P, will be the least, that A, B, C, measure; for if it be possible let there be another R, less; that A, B, C, measures; therefore P, is not the least that A, B, measure, which is against the Hypothesis.

A, B, C,
P, R,
X.

a 36. 7.

But if C does not measure P, find *b* R the least that P and C measure, and R shall be the least, that A, B, C measure, for because, A, B measure P, and P. measures R, also A and B shall measure *c* R. but C also measures the said R, therefore the three Numbers A B C measure R.

b by the same.
c by the præc.

And that R. is also the least is thus shewn, if it be possible let X, be less than R, which A, B, C, measure, therefore P the least that A. B measure, shall also measure *d* X, therefore seeing C, P, measure X (C. by the Hyp. and P. by what is already demonstrated) a number less than R; R shall not be the least which C, P measure, which is absurd and against the Construction.

d By the præc.

Cor.

Corollary.

1. BY the ſame artifice, if Numbers how many ſoever are given, the leaſt which they all meaſure, may be found.

2. If three Numbers A, B, C, (or more) meaſure any other number X, alſo R, the leaſt which A, B, C, meaſure, ſhall meaſure X.

The Conſtruction being made as above, ſeeing A, B, *a* meaſure R; alſo P *b* ſhall meaſure X, and ſeeing C, P, meaſure X. (C by Hyp. and R, by what is already demonſtrated) alſo R ſhall meaſure X. W. W. demonſtrated.

a Hyp.
b 27. 7.

PROP. XXXIX.

IF a number (A) meaſureth another number (B) by the quotient (C), the quotient ſhall be a part of the meaſured (B,) denominated from the meaſuring (A)

For ſeeing A meaſureth B by C, alſo C ſhall meaſure *a* B by A, (that is C being ſo often taken as there are Units in A,) maketh B, and ſo C, is a part of B. denominated from A.

A. 3. B. 24. (C 4

a ax. 9.

PROP. XL.

IF a Number (A,) hath any part whatſoever (B.) the Number (E) denominating the part, ſhall meaſure the ſame.

Becauſe B. is a part of the Number A denominated from E therefore B meaſures A by E, and therefore again E, meaſureth *b* A by B.

A 24 B. 6. E. 4.

b ax. 9.

PROP.

PROP. XLI.

TO find the leaſt Number which hath or containeth the parts denominated from the Numbers A. B, C. given.

Find *a* G the leaſt which the given denominators A, B, C. of the parts, meaſure, I ſay that G is the number ſought. *a* 38. 7.

A. 2. B. 3. C. 4.
G. 12.

For A, B, C meaſure the ſaid G. by the Quotients P. Q. R. therefore P, Q, R, are *b* parts of the ſaid G. denominated by A, B, C; but that G. is the leaſt is manifeſt; for if X leſs then G, hath parts denominated from the Numbers A, B, C; A, B, C, *c* ſhall meaſure the ſaid X, and therefore G would not be the leaſt which they meaſure, contrary to the Hypotheſis. *b* 39. 7. *c præc.*

P. 6 Q. 4. R. 4.
X.

Corollary.

THE leaſt number which numbers A, B, C, how many ſoever given meaſure, is alſo the leaſt of all that have parts denominated from the ſaid given numbers.

THE

THE SECOND BOOK OF THE ELEMENTS OF Arithmetick, And the Eighth of EUCLID.

PROP. I.

IF there be divers Numbers, how many ſoever (A, B, C, D) *whoſe extreams* A, D, *are prime the one to the other, then thoſe Numbers* A, B, C, D, *are the leaſt of all in the ſame Proportion with them.*

For if it be poſſible let there be as many others E, F, G, H. leſs and Proportional to the ſaid A, B, C, D.

A, B, C, D.
E, F, G, H.

there-

therefore by Equality of Reaſon (a) A . . D :: E . . H. And becauſe A and D prime one to the other, they are the (b) leaſt in the ſame Proportion with them; and conſequently will meaſure the ſaid (c) E H leſſer then themſelves, which is abſurd.

a 12. 5.
b 23. 7.
c 21. 7.

Euclid *makes the Numbers* A, B, C, D, *to be in continual Proportion, which does not appear by the Demonſtration to be requiſit.*

PROP. II.

TO find out the leaſt Numbers continually Proportional, as many as ſhall be required, in the Proportion given of A *to* B.

Let A, B, be the leaſt Terms of the given Ratio, now A x A = A A A x B = A B, B x B = B B. then AA AB, BB, are the three leaſt Terms in the Proportion of A to B.

Unity.			
A 2,	B 3.		
AA,	AB,	BB,	
4	6	9	
AAA,	AAB,	ABB,	BBB.
8	12	18	27

That they are in the ſame Propoſition as A to B is manifeſt from Prop. 17. 7. and its Scholium. And that they are the leaſt I thus ſhew; becauſe A, B, are the *a* leaſt in their Proportion, they are *b* prime the one to the other; wherefore alſo their Squares AA, and BB, are *c* prime to one another; therefore AA, AB, BB, are the three *d* leaſt Terms in their Proportion; that is in the Ratio of A to B.

a Hyp.
b 24 7.
c 29. 7.
d prac.

Then A multiplying the three Numbers already found makes AAA, AAB, ABB, and B multiplying the third BB, makes BBB, theſe four are the leaſt Proportionals in the Ratio of A to B; But that they are Proportionals in the Reaſon of A to B, appears from 17 *E.* 7. and its Scholium; and that they are the leaſt is manifeſt from 29 *E.* 7. and from what goes before.

For

For AAA, BBB, are (e) prime the one to the other, are consequently AAA, AAB, ABB, BBB, are the four (f) least in their Proportion, which is as A to B.

(e) 29. 7.
(f) *prec.*

In the same manner may be found four, five six, continual Proportionals, and so on in infinitum, the their Proportion.

Corrollarys.

1. HEnce if three Numbers AA, AB, BB, being the least are continually Proportional, their extreams are Squares, if four, their extreams are Cubes and so on.

2. The Extreams of Numbers found to be the least, and in continual Proportion, are prime the one to the other; this is manifest from their Genesis and from 29. and 7.

3. Two Numbers. A B being the least of a given Ratio, measure all the other Terms, altho they are infinite, for seeing A, B generate all the rest; they will also measure them by *Axiom* 7. and *Corral. Prop.* 16. E. 7.

4. Unity, A, AA, AAA, are continually Proportional, or continual Proportionals; likewise also, Unity B BB BBB are, for seeing A x A = AA, it will be (g) 1 . . A :: A . . AA and seeing A x AA = AAA, it will be (h) 1 . . A :: AA . . AAA. therefore A . . AA :: AA . . AAA.

(g) Def. 13.
(h) *by the same.*

5. Between the Extreams AAA and BBB, there fall as many means as between them and Unity, which is manifest from what is already Demonstrated.

PROP. III.

IF there are continual Proportionals, A, B, C, D, how-many soever, being also the least of all that have the same Proportion with them, their Extreams A, D, are prime to one another.

a Find

a Find two Numbers E F, which are the leaſt in the Ratio of A to B, then by the preceeding find ſo many, O P Q R, the leaſt in the Ratio of E to F; therefore becauſe both O, P, Q, R. as well as A, B, C, D, are the leaſt in the Ratio of E to F, and as many as one another; they muſt of neceſſity be one and the ſame Numbers. But the Extreams O, R, are *b* prime to one another; therefore alſo A, D are prime to one another. W. W. Demonſtrated.

A, B, C, D.
E, F,
O, P, Q, R.

a 53. 7.

b Cor. 2. *præc.*

Hence a Series of continual Proportionals being in their leſt Tearms, and whoſe extreams are conſequently prime the one to the other, cannot be continued further, as ſhall be demonſtrated in Prop. 17. 9.

PROP. IV.

PRoportions or Ratios, how many ſoever being given in their leaſt Numbers, to continue them alſo in the leaſt Numbers.

1. Let there be given two Ratio's, A to B, and C to D, in their leaſt Terms; find *c* O the leaſt Number which B and C meaſure; then ſo often as B meaſures O, ſo often let A meaſure the Number N; and ſo often as C meaſures O, let D meaſure P; I ſay that N O, P, are the leaſt that continue the given Ratio's, A to B, and C to D; and that they do continue the given Ratio's is manifeſt from their Geneſis, by reaſon whereof A . . N :: B . . O, and C . . O :: D . . P. wherefore by permutation A . . B :: N . . O, and C . . D :: O . . P; and that they are the leaſt, I thus ſhew; For if it be poſſible let Q, R, S, leſſer than the former continue the given Ratio's; wherefore becauſe A . . B :: Q . . R, and that A B are the leaſt in their Reaſon, it is manifeſt that *d* A, B, meaſure the ſaid Q R, for the ſame Reaſon C, D will meaſure R, S; wherefore becauſe both B and C meaſure R alſo O, the leaſt

A . . B . C . . D.
N. O. P.
Q. R. S.

c 36. 7.

d 21. 7.

which B and C measure, *e* shall measure the said R, the greater the lesser, which is absurd.

e 37. 7.

2. Let there be given three Ratios in their least Tearms, A to B, C to D, E to F, find O the least, which B and C measure; and so often as B, C measure O, so often let A, D, measure the Numbers N P; if then also E measures P, as often as F measures Q, I say, that N.O,P,Q are the least that continue the given Ratios or Proportions.

A. B. C. . D. E. . F
N. O. P. Q.
R. S. T. V.

But that N . . O :: A . . B. and O . . P : : C . . D, and P . . Q : : E . . F, appears from the construction its self; and that they are the least I thus shew; If it be possible let others lesser R, S, T, V, continue the given Ratios. I will shew as above that the greater O measures the lesser S, which is absurd.

But if E does not measure P, find the least S, which P and E measure, and so often as P measures S, so often let O, N, measure the Numbers R, Q; also so often as E measures S, so often let F measure T: I say, that Q. R. S. T- are the least which continue the given Ratios.

A . . B . C . . D . E . . F
N . O . P.
Q. R. S. T.
V—X—Y—Z

For by Construction it is manifest that S . . T : : E . . F, also that Q, R, S are proportional to the said N, O, P; but N, O, P, continue (as is shewn above) the Ratios A . . B and C . . D; therefore also Q, R, S continue the same, and consequently seeing that E . . F : : S . . T, it is manifest that Q, R, S, T. continue the three Reasons A . . B; C . . D; E . . F; but that Q. R, S, T are the least, I thus shew; if it be possible let others less, V, X, Y, Z continue the three Ratios given, wherefore because A . . B : : V . . X, and that A, B are the *o* least in their Proportion, B *f* will measure X, after the same manner it may be shewn that C measures X, therefore also *n* O the least which B and

o Hyp.
f 21. 7.
n 37. 7.

and C meaſure, will meaſure the ſaid X: now becauſe X..Y::C.,D, that is O..P, alſo by Permutation X..O::Y..P, therefore becauſe O meaſures X, alſo P will meaſure Y; but alſo E will meaſure Y (ſeeing E, F are *g* the leaſt in their Proportion; and that E..F::Y..Z) therefore alſo S, the leaſt which E and P meaſure, will meaſure Y, the greater the leſſer, which is abſurd. *g* H, p.

By the ſame Artifice four Ratios, or more, in infintum, may be continued in their leaſt Terms.

PROP. V.

THe proportion, or ratio of plain numbers (AB. CD) the one to the other is compoſed of the ratios of their ſides; that is of A..C, and B..D; or of A..D, and B..C.

AB. CD. BC.

For B x C=BC, the Ratio of AB to CD is compoſed of the Ratios AB..AC and BC..CD, for AB..AC::BC..CD; but the Ratio AB..BC is=*a* to the Ratio A..C and the Ratio BC..CD=*b* Ratio B..D, therefore alſo the Ratio AB..CD is compoſed of the Ratios of the ſides A..C, and B..D W. W. Demonſt. *a* 17. 7. *b ibid.*

PROP. VI.

IF Numbers how many ſoever are in continual proprotion A, B, C, D, E, *and the firſt* A, *doth not meaſure the ſecond* B, *neither ſhall any of the other meaſure any one of the reſt.*

That none meaſures the next following is manifeſt from the hypotheſis its ſelf, but that any other doth not meaſure any one of the reſt, I thus ſhew.

Let there be found *a* N, O, P, the leaſt in the ſameproportion with A, B, C, thefore N, P, are prime the one to the other and by *b* equallity it will be A..C::N..P. but becauſe A..B::N..O, and that

A, B, C, D, E.
N, O, P.

a 33. 7.
b 22. 7.

A does not measure B, neither will N measure O, and consequently N is not a Unite.

A, B, C, D, E, N, O, P

Wherefore seeing N, P are prime the one to the other, N, will not *c* measure P, but A . . C :: N . . P.

c def. 22.

wherefore A doth neither measure C, after the same manner I will shew that B doth neither measure D, the third number from its self, nor C, the third E from its self; and if four A, B, C, D, continual proportionals the least in their proportion be assumed, it may be demonstrated by the like way, that neither A doth measure the fourth D, nor B the fourth from it E, and so on.

PROP. VII.

IF *there are Numbers, how many soever* A, B, C, D, E, *in continual proportion and if one of them measures any other; (not the second* B,) *also the first* A, *shall measure the second* B.

For if A does not measure B. neither shall any of the other *a* measure any of the rest, contrary to the hypothesis.

a prec.

PROP. VIII.

IF *four Numbers are in the same proportion* (A . . B :: C . . D) *as many mean proportionals as falls between the two first* (A *and* B) *so many shall also fall between the two last* (C *and* D)

Between A, B, let the means P. Q. A, P, Q, B, fall, and find the *a* least proportionals N, O, R, S, N, O, R, S. to the numbers A, P, Q, B, C, V, X, D, therefore by *b* Equality it will be N . .

a 5, 7.

b 22, 5.

c 3, 8. S :: A . . B that is C . . D; but N, S, are *c* prime the one to the other, and consequently the *d* least in their proportion, therefore N, S, equally measure *e* C, D proportional to them, as often as O and R, measure others V, X, wherefore because N, O, R, S, equally

d 22, 7.

e 21. 7.

equally meaſure the ſaid C, V, X, D. it is manifeſt that C, V, X, D, are proportional to the ſaid N, O, R, S, that is to *f* A, P, Q, B given, wherefore ſeeing A, P, Q, B are continual proportionals, alſo C, V, X, D are ſo many continual proportionals and conſeqnently between A, and B, and C and D, there fall equal Number of mean proportionals, W. W. Demonſtrated.

f Hyp.
g Hyp.

PROP. IX.

IF two Numbers C and D, are prime to one another, as many mean proportionals as fall between them, ſo many means fall between each of them and a Unite.

Between C and D let there be the mean proportionals O, P. find *a* A, B, the two leaſt in the Ratio of C to D, then the three AA, AB, BB, laſtly the four AAA, AAB, ABB, BBB, untill the multitude of thoſe found be equal to the multitude of the given C, O, P, D wherefore becauſe the extremes C, D, are *b* Prime one to the other, C, O, P, D are *c* the leaſt in their proportion, that is, the leaſt in the ratio *d* of A to B; wherefore ſeeing AAA, AAB, ABB, BBB are alſo *e* the leaſt in the Ratio of A to B, theſe ſhall be equal to them, each to each; laſtly from the 2 Prop. of this Book, and def. 14. it is manifeſt that Unity and A, AA, AAA, alſo Unity and B, BB, BBB, are continually proportional. Wherefore ſeeing that the number of terms AAA, AA, A, as well as BBB, BB, B, with Unity is equal to the number of terms AAA, AAB, ABB, BBB, that is C, O, P, D, as many means as fall between C and D, ſo many fall between a Unite and AAA or C; alſo between a Unite and BBB or D. which was to be demonſtrated.

C,	O,	P,	D
	1.		
	Unity		
A			B
AA,	AB,		BB
AAA,	AAB,	ABB,	BBB

a 2, 8.
b Hyp.
c 23, 7.
d Hyp.
e conſtr.

PROP. X.

IF between a Unite and two Numbers (AAA and BBB) there fall an equal number of mean proportionals: there alſo fall the ſame number of means between the numbers AAA, and BBB themſelves.

Let the whole conſtruction of Prop: 2. of this Book be conſidered, and the Demonſt. of this Prop. will be manifeſt from the 4 and 5 Corollaries of the ſaid Prop.

Corollary.

Unity	
1.	
A	B
AA, AB, BB	
AAA, AAB, ABB BBB.	

1. IF there are two ranks of continual proportionals, from Unity 1, A, AA, AAA, *&c.* 1. B, BB, BBB, *&c.* The Ratio of AA to BB, will be duplicate to that of A to B, and the Ratio of AAA to BBB will be triplicate of that of A to B and ſo on

a Sch. p. 17, 7. For ſeeing that AA, AB, BB, are continual *a* proportionals in the reaſon of A to B, therefore the Ra-
b def. 12. tio AA .. BB is dublicate of the *b* Ratio AA .. AB,
c Sch. 17, that is of A .. B; likewiſe ſeeing AAA, AAB, ABB, BBB are continually proportional in the Ratio of A
d def. 12. to B, the Ratio AAA .. BBB ſhall be triplicate of the Ratio *d* AAA .. AAB. that is of A .. B, and ſo on W. W. D.

2. The firſt Corollary will hold true, let the ranks or ſeries begin from any common number C.

A	
B. E.	
C. O. F.	
D. P. Q. G.	

For ſeeing that A, B, C and A, E, F are in continual proportion, therefore CA = *a* BB, and FA = EE, but the Ra-
a 20, 7. tio of BB .. EE is duplicate of the Ratio B .. E by the 11 which does not depend on this, therefore alſo the Ratio
b Sch. 17, CA .. FA (that is the ratio *b* C to F) is duplicate of the

the Ratio B . . E, which was the first, in like manner I will shew, that the ratio DB . . GE is dupricate of the ratio C . . F, and consequently quadruplicate of the ratio B . . E; but the ratio DB . . GE, is composed of the ratios D . . G and B . . E, therefore the ratio D . . G with the ratio B . . E is quadruplicate of the ratio B . . E. therefore the sole ratio D . . G is triplicate of the ratio B . . E. which is the other.

3. If between the number A, and the two numbers D. G there fall equal number of mean proportionals B, C, E, F. also between the said D, G (although either of them be a Unite) there will fall the same number of means.

This is manifest from Corol. 2. for seeing the ratio D . . G is triplicate of the ratio B . . E, between D and G there falls two *c* mean proportionals in the ratio B . . E. *c def.* 12.

PROP. XI.

BEtween two square numbers (AA, BB) there falls one mean proportional (AB) and the proportion of a square number to a square number is duplicate of the proportion of the sides (A, B)

Part. 1, AA . . AB : : *a* A . . B and AB . . BB : : A . . B therefore AA . . AB :: AB . . BB which was the first. AA, AB, BB A. B. *a Scho.* 17. 7. 0. 11. 5.

Part. 2. From the first Part, it is manifest that the ratio AA . . BB is duplicate of the *b* ratio AA . . AB; that is of the ratio *c* A . . B. *b def.* 12. *c Sch.* 17. 7.

SCHOLIUM.

FRom this, and the 8 Prop. aforegoing may be Demonstrated, that Celebrated Theor. *which is the last of the Tenth of* Euclid; in a square the diameter is incommensurable to the side.

For if it be denied, the diameter is to the ſide, as a number is to a number, Suppoſe as a *to* b, *as is manifeſt from the def. of commenſurable magnitudes, therefore alſo (as is manifeſt from* 22 *Eu.* 6.*) the ſquare of the Diameter is to the ſquare of the ſide, as a ſquare number* aa *is to a ſquare number* bb, *which cannot be, for ſeeing (as is manifeſt from* 47. *Eu.* 1.*) the ſquare of the Diameter is to the ſquare of the ſide as* 2 *is to* 1. *If that were to this as the ſquare number* aa *is to a ſquare number* bb, *alſo it would be* aa . . bb : : 2 . . 1 *and conſequently ſeeing between* aa *and* bb *by this Prop. there falls one mean, a whole number, alſo between* 2 *and* 1 *there would fall one integral mean by the* 8 *Prop. of this Book which is abſurd.*

PROP. XII.

BEtween two Cube Numbers (AAA and BBB) there falls two mean proportionals (AAB, ABB) and the proportion of a Cube to a Cube, is triplicate of the proportion of their ſides A, B.

AAA, AAB, ABB, BBB.
A, B,

By the Scholium to Prop. 17. 7. AAA, AAB, ABB, BBB are continual proportionals in the reaſon of A to B, which was the firſt; from which and the 12 Def. the ſecond is manifeſt.

PROP. XIII.

IF there be given Numbers, how many ſoever, A, *B, C, in continual proportion, which being multiplyed in themſelves make AA, BB, CC, and being again multiplyed in theſe make, AAA, BBB, CCC and ſo on in Infinitum.*

I ſay that AA, BB, CC, and alſo AAA, BBB, CCC, and ſo on; are continually proportional. By Corol. 4. Prop. 2. of this Book, three ranks here beginning from Unity are continual proportionals. Therefore the

the ratio AA.. BB *a* is duplicate of the ratio A.. B, that is of the *b* ratio B.. C but alſo the ratio BB.. CC is duplicate *c* of the ratio B.. C. Therefore *d* AA.. BB :: BB.. CC; in like manner becauſe the ratio AAA.. BBB is triplicate *e* of the ratio A .. B that is of *f* B.. C, whoſe triplicate is alſo *g* the ratio of BBB.. CCC, alſo it will be *h* AAA.. BBB :: BBB.. CCC, which was to be demonſtrated.

1	1	1	
A	B	C	
AA	BB	CC	
AAA	BBB	CCC	

a cor. pr. 10. 8.
b Hyp.
c cor. pr. 10. 8.
d 34. 5.
e cor. pr. 10. 7.
f Hyp.
g cor. pr. 10. 7.
h 34. 5.

PROP. XIV.

IF a Square Number (AA) meaſure a ſquare Number (BB,) alſo the ſide (A) of one, ſhall meaſure the ſide (B) of the other.

And if the ſide A, meaſure the ſide B, alſo the ſquare ſhall meaſure the ſquare.

Part, 1. A x B = AB, by Schol. Prop. 17, Eu. 7. AA. AB. BB. are proportionals in the Ratio of A to B; wherefore, ſeeing *a* AA meaſures BB, alſo *b* A ſhall meaſure AB; But AA.. AB :: *c* A.. B, therefore alſo A meaſures B, which was the firſt.

AA.	AB.	BB.
A.	B.	

a Hyp.
b. 7. 8.
c Sch. p. 17. 7.
d ax. 11.

Part 2. A .. B :: *e* AA .. AB, but A is already put to meaſure B. therefore alſo AA. meaſures AB, again, becauſe A .. B :: *f* AB .. BB, and that A *g* meaſure the ſaid B, alſo AB. ſhall meaſure *h* BB. therefore alſo AA ſhall meaſure BB, which was the other.

e Sch. p. 17. 7.
f ibid.
g Hyp.
h ax. 11.

PROP. XV.

IF a Cube number AAA, meaſures a Cube number BBB. alſo the ſide A ſhall meaſure the ſide B, and the contrary.

Part

Part 1. Between both the Cubes, ſet the numbers AAB, ABB, whoſe Geneſis the Letters ſufficiently declare; all four are in *a* continual proportion, in the reaſon of A to B. wherefore ſeeing AAA meaſures BBB, *b* alſo AAA ſhall meaſure *c* AAB, but AAA.. AAB:: *d* A.. B. therefore alſo A. meaſures B. which was the firſt.

AAA, AAB, ABB, BBB.
A. B.

a Sch. 17. 7.
b Hyp.
c 7. 8.
d Schol. 17. 7.

Part 2. Becauſe AAA.. AAB:: *e* A.. B, and that A is put to meaſure B, alſo AAA. ſhall meaſure AAB; in like manner, becauſe AAB. ABB. BBB. *f* are one unto another, as A is to B. alſo AAB ſhall meaſure ABB, and ABB, the ſaid BBB, therefore alſo *g* AAA meaſures BBB, which was the other.

e ibid.
f ibid.
g ax. 11.

PROP. XVI.

IF the ſquare (AA) doth not meaſure the ſquare (BB) neither ſhall the ſide (A) meaſure the ſide (B,) and the contrary.

Part 1. For if the ſide A, doth then meaſure the ſide B; alſo AA, *a* would meaſure BB. contrary to the Hypotheſis.

AA. BB.
A. B.

a 2 *part pro.* 13. 8.

Part 2. If the ſide A not meaſuring B, AA meaſures the ſaid BB, it would alſo follow that *b* A would meaſure B. contrary to the Hypotheſis.

b 1 *part of the ſame.*

PROP. XVII.

IF the Cube AAA, does not meaſure the Cube DDD; neither ſhall the ſide A meaſure the ſide D, and the contrary.

The is demonſtrated from the abſurdity of its contrary by Prop. 15.

PROP.

PROP. XVIII.

BEtween two like plain Numbers (AB, CD) there is one mean proportional Number, and the Ratio of a plain Number to a plain, is duplicate of the ratio of their Homologus ſides (A, C.)

Part 1. Becauſe AB, CD are like Plaines, it will be *a* A..B::C..D and by permutation A..C:: B..D; ſceing therefore that AB..BC:: *c* A..C; that is (as I have already ſhewn) B..D; that is *d* BC..CD, BC ſhall be a mean Proportional betwen AB, & CD, which was the firſt.

AB, BC, CD, A. B. C. D.

a Def. 27. *b* 16 5 *c* Schol. 17 7. *d* ibid. *e* defin. *f* Schol. 17 7.

Part 2. By the firſt part, the Ratio of A B to C D is Duplicate of the *e* Ratio of AB to BC; that is of the Ratio of A to C, which was the other.

PROP. XIX.

BEtween two like Solids (A B C, D E F) there falls two mean Proportionals (BCD, CDE) and the Ratio of a Solid to a Solid is Triplicate of the Ratio of their Homologus ſides.

Part 1. Becauſe the Solids ABC, and BEF, are ſimular, or like the ſides; A B, C of one are Proportional to the ſides D, E, F, of the other, wherefore by Permutation A..D:: B..E and C..F. now by the Schol. to Prop. 17. Eu. 7. ABC.. BCD:: A..D; BCD..CDE:: B..E; CDE..DEF:: C..F. but I have already ſhewn that A..D:: B..E:: C..F. therefore alſo ABC.. BCD:: BCD.. CDE:: CDE.. DEF, and conſequently BCD, CDE are two mean Proportionals, between ABC, and DEF, which was to be Demonſtrated.

ABC. BCD. CDE. DEF. A. B. C. D. E. F.

Part

Part 2. This is already manifeſt from the firſt part, and the 12 Def. *viz.* That the Reaſon of ABC to DEF is triplicate of the reaſon of ABC to BCD; that is *a* of the Reaſon of A to D, which was to be demonſtrated.

a Scholi. 17 7.

Scholium.

THis place requires that we demonſtrate, that when a Solid Number is produced of three Numbers, or of Numbers how many ſoever, multiplied any how the one by the other, they do always produce the ſame Number.

This is an excellent Theorem, and of great uſe, which we will demonſtrate otherways, and far more expedite then has been done by others; and becauſe it depends on the permutations which can be made of a given Number of things; we will premiſe this.

Theorem 1.

LET there be given Things or Letters, A, B, C, D, E, as many as you pleaſe; and take ſo many Numbers from Unity 1, 2, 3, 4, 5, *as there are Letters, which multiply one into the other orderly, and the Product ſhall be the Number of Permutations, which can be made of the given things, A, B, C, D, E.*

Two Letters (which we aſſume for things) a, b, may be twice changed, each ſtanding the firſt or laſt place once; hence,

Three Letters a, b, c, may have ſix permutations, for any one of the three occupying the third place, the other two may be changed twice, for when c is in the laſt place, the other two a b, may be changed twice; and ſo there will be had the two divers orders, abc, bac. Again if b is in the laſt place, the others a c, may be twice changed, and ſo there will ariſe the two new Orders, acb, cab: laſtly, if a be in the laſt place, the remaining b c, have two per-

permutations, whence again arise two other orders, bca, cba: see them together.

abc	acb	bca
bac	cab	cba

Four Letters, a, b, c, d, have 24 permutations, for any of the four Letters may once be in the last place, and then the other three may be permuted 6 times, whence there will arise 24 several Otders of the four Letters.

Five Letters, a, b, c, d, e, may have 120 permutations; for any of the five being in the last place, the other four may be permuted 24 times, whence there arise 5 times 24 several orders of the 5 Letters given, which is 120, and so in infinitum. This being premised, we proceed to

Theorem 2.

Part, 1. *THree Numbers A, B, C, being in any order, multiplied the one by the other, always produce equal Numbers.*

The Table here set shews six several Orders of Multiplications. That the Products are equal when the same Letter is in the least place, as in a B c and b a c, &c; is manifest from 16 E. 7. Therefore it remains to shew that a B c, a C b, b C a, are equal.

a B c
b a c
a C b
c a b
b C a
c b a

Let us compare the first a B c and a C b; a B.. *a* a C:: B..C but B and C are the same numbers as b and c, therefore a B..a C::b..c; therefore *b* a B x c (= a B c) = a C x b (= a C b;) Now let us compare a B c and b C a, a B

a B c
a C b
a B c
b C A

..b C

∴ b C :: a .. C; that is A .. C; therefore the Product a B c = b C a, therefore it is manifeſt that all the ſix Products are equal to one another.

Part 2. *Likewiſe 4 Numbers, a, b, c, d, being multiplied in any order, the one into the other, produce equal Numbers.*

By Theor. 1. It is manifeſt that a, b, c, d, hath 4, Senaries of different Orders, that is 24 different Orders; the firſt Senary is exhibited in the appoſed Table; becauſe a, b, c, multiplied the one into the other any way, always produce the ſame Number by the firſt part hereof; and the laſt in the whole Senary is the ſame; it is manifeſt that all the Products of the firſt Senary are one and the ſame, or equal: After the ſame manner may be ſhewn that the Products of the ſecond Senary agree; and the like may be demonſtrated of the third and fourth Senary. Now this only remains, that the Product of one Senary agreeth with any other of the three remaining Senaries, which I thus ſhew;

abcd
bacd
acbd
cabd
bcad
cbad

Let us compare the firſt Senary in which d is in the firſt place, with the fourth Senary, in which a is in the laſt place; write a under d, then before d put A, and D before a, and ſet the other Letters bc, before both; therefore bcA .. bcD :: A .. D; that is a .. d; therefore the Products bcAd, and bcDa of the firſt and fourth Senary are equal. In like manner I will ſhew that the Products of the 4 Senarys agree, therefore the propoſed is manifeſt.

bcAd
bcDa

Part 3. *By the ſame Ratiocination by Part 2. may be ſhewn, that the Products ariſing from the Multiplication of 5 Numbers, which are 120, are the ſame, and ſo on in infinitum in 6, 7, 8 &c. Numbers.*

PROP.

PROP. XX.

IF between two plain Numbers (A and B) there falls one mean Proportional C, they are like plain Numbers.

Aſſume F,K *a* the leaſt in the ratio of A..C and C..B; then *b* F,K equally meaſure as well the ſaid A, C. ſuppoſe by X; as the ſaid C, B, ſuppoſe by Z, therefore *c* K x X = C and *c* K x Z = B, and conſequently X..Z :: *d* C..B :: F..K, and by permutation F..X :: K..Z. wherefore ſeeing *e* F x X = A, and K x Z = B, and that conſequently F, X, and K, Z, are the ſides of the Planes A and B, therefore A, B are like *f* plain Numbers, which was to be Demonſtrated.

A. C. B.
F K
X Z

a 35 7
b 21 7
c *Ax.* 8
d 17 7
e *Hyp.* & *Ax.* 8
f *Def.* 27

Something otherways.

IF between two Numbers, (A, B) there falls one mean Proportional C, they are like plain Numbers.

Take *a* D, E the leaſt in the ratio A..C and C..B, therefore D, E equally meaſure as well A, C, ſuppoſe by M, as C B ſuppoſe by N. wherefore D x M = D M = *c* A, and E M *d* = C, and *e* E N = B; now in D M and E M; D..E :: *f* D M..E M: :A..C :: *g* C..B :: E M..E N :: *h* M..N; therefore D M, E N, that is A, B, are *i* like plaine Numbers which was to be Demonſtrated.

A. C. B
D E
M N
DM. EM. EN.

a 35 7
b 21 7
c *Ax.* 8
d *the ſame*
e *the ſame*
f *Sco.* 17 7
g *Hyp.*
h *Schn.* 17 7
i *Def.* 27

PROP

PROP. XXI.

IF between two Numbers (A, B) *there fall two mean Proportionals* (C, D) *the Numbers* (A, B) *are like ſolid Numbers.*

a 35 7 Let there be taken E P, X, O Q, the *a* three leaſt proportionals to A, C, D

b Cor. 21 7 which may alſo equally *b* meaſure the ſaid A, C, D. ſuppoſe by M; and conſequently E P M (= E P x M) = *c* A and X M = *d*

A. C. D. B
EP X OQ
M N
EPM, XM, XN OQN.

c Ax. 8

d the ſame C. but becauſe C, D, B, are in the ſame reaſon, as A, C, D; thence EP, X, OQ are the leaſt Proportionals to the ſaid C, D, B, and conſequently equally

e Cor. 21. 7 *e* meaſure them, ſuppoſe by N, wherefore XN (= X x N) = *f* D and OQN = B. Seeing therefore, that EPM = A and OQN = B, it remaineth to ſhew, that the ſides E, P, M are Proportional to the ſides O, Q, N.

f Ax. 8

g Conſtr. *h prec.* *i Conſtr.* *k Sc.* 17. 7. *l Conſtr.* *m Def.* *n* 18 8 *o* 11 12 *p Def.* 27. Becauſe between EP and OQ there is a *g* mean Proportional X, the ſaid EP, OQ *h* are like Plains, therefore *i* E .. O :: P .. Q; then M .. N :: *k* XM .. XN; (that is as I have already ſhewn above) C .. D that is *l* EP .. X, but EP .. X :: P .. Q; becauſe both *m* EP .. X and *n* P .. Q are in half the Ratio of EP .. OQ, therefore *o* P .. Q (that is E .. O) : : M .. N: And therefore the Solids EPM, OQN, that is A, B are *p* like Solid Numbers. W. W. to be demonſtrated.

PROP.

PROP. XXII.

IF three Numbers (A, B, C) are Proportional, and the first (A) be a Square, the third also shall be a Square.

Let the root or side of the Square A be O; because therefore, O x O = A, it will follow, 1 .. O :: *a* O :: A; and consequently both between A and 1, as also between A and C, there falls one mean Proportional; therefore also *b* between 1 and C falls a mean Proportional P, which being multiplied into its self *c* makes C, therefore also C is a *d* square Number. W. W. Demonstrated.

A: B. C.
O P
1

a Def. 13
b Cor. 3. 10. 8.
c Def. 13
d Def. 28

Corollary.

A Square, the Root, and Unity, are continual Proportionals; which appears from the Demonstration aforegoing.

PROP. XXIII.

IF of four Numbers (A, B, C, D) continually Proportional, the first (A) be a Cube, also the fourth (D) shall be a Cube.

Let the side of the Cube A, be O, then O x O = P, and O x P = *a* A the Cube; now by *Cor.* foregoing 1 .. O :: O .. P, and because O x P = A, it will be 1 .. O :: *b* P .. A; therefore both between A and 1, and between A and D, there fall two mean Proportionals; therefore also between 1 and D, there *c* falls two means, Q, R; seeing therefore 1 .. Q :: R .. D, it

A, B, C, D
P R
O Q
1

a Def. 29
b Def. 13
c Cor. 3. 10. 8
d Def. 13

e Def. 13 it is manifeſt that *e* Q x R = D. Therefore alſo *f* D
f Def. 29 is a Cube Number.

Corollary.

UNity, the Root, the Square, and the Cube, are continual Proportionals, which is manifeſt from what is Demonſtrated.

PROP. XXIV.

IF two Numbers (A, B) have the ſame Proportion one to the other, as a ſquare Number C, hath to a Square D, and if the firſt A be a ſquare, the ſecond B, ſhall alſo be a ſquare Number.

Becauſe A . . B : : C : . D, and between C and D there falls one mean
A. O. B
C. P. D
a 11 8
b 8 8
c 22 8
P a Proportional P, alſo between *b* A and B, there falls one mean O, becauſe therefore the firſt A is a Square, the ſecond alſo *c* B ſhall be a Square. W. W. Demonſtrated.

PROP. XXV.

IF two Numbers (A, B) have the ſame reaſon the one to the other, as a Cube C hath to a Cube D; and if the firſt A be a Cube, the ſecond B alſo ſhall be a Cube Number.

Becauſe A . . B : : C . . D, and that between the Cubes *a* C, D, there fall two mean Proportionals Q, R; alſo between A, B, *b* will fall two means; now becauſe A is a Cube,

a 12 8
A. O. P. B
C. Q. R. D
b 8 8

Cube, therefore also B will be *c* a Cube. W. W. *c* 23 8
Demonstrated.

Corollaries.

1. THE Ratio of a square Number to a Number that is not a square, cannot be expresst in two squares; as is evident from Prop. 24.

2. The Ratio of a Cube Number to a Number that is not a Cube, cannot be expresst in two Cubes; as is manifest from Prop. 25.

PROP. XXVI.

LIKE plain Numbers (A, B) have the same Proportion the one to the other, as a square Number hath to a square Number.

Between A and B there falls one *a* mean Proportional, which let be C, find the three least Terms D, E, F, in the Ratio's of A .. C or C .. B the Extreams D, F, are *b* Squares; because A .. C :: *c* D .. E and C .. B :: *d* E .. F: Therefore by equality *e* A .. B :: □ D .. □ F. which was to be demonstrated.

a 18 8

A. C. B.
D. E. F.

b *Cor.* 1 2 8
c *Constr.*
d *Constr.*
e 22 7

PROP. XXVII.

LIKE Solid Numbers, (A, B) have the same Reason the one to the other, as a Cube to a Cube Number.

a 19 8 Between A and B there falls two *a* mean Proportionals, which let be C, D;
A. C. D. B find the four least Terms, E, G,
E. G. H. F H, F, in the Ratio of A. . C;
b Cor. 1 2 the Extreams E and F are *b* Cubes; and by equallity it will be A. . B:: Cube E. . Cube F. W. W. Demonstrated.

THE

THE THIRD BOOK OF THE ELEMENTS OF Arithmetick, And the Ninth of EUCLID.

PROP. I.

TWO like Plains (*A*, *B*,) *being multiplied the one into the other, produce a ſquare Number* (*AB.*)

A xA = AA, therefore it will be AA . AB :: A . B, but between *b* A and B, there falls one mean

a Sch. 17. 7
b 18 8

A: O. B.
c 8 8 AA. P. AB

Proportional, seeing A and B are like Plains: Therefore also between *c* AA and AB will fall the mean P; wherefore seeing the first AA is a Square by Construction, also the third AB shall be a Square; which was to be Demonstrated.

PROP. II.

IF two Numbers (A, B) *being multiplied together, produce a Square* (AB) *those two Numbers are like plain Numbers.*

A x A = AA, because AA, and AB are both Squares, there falleth a *a* mean Proportion between them, but AA..

a 11 8
A. O. B.
b Sch, 17. 7 AA. P. AB.
c 8. 8
d 20. 8

BB:: *b* A.. B; therefore also between A and B, there falls a *c* mean Proportional. Therefore A, B, are *d* like plain Numbers. W. W. D.

Corollaries.

1. TWO Squares produce a Square, for they are like Plains, therefore by the first hereof.

2. If two Numbers A, B, produce a Square, and one of them (suppose A) be a Square, the other also B shall be a Square

For by the Second of this Book, A and B are like
a 18 8 Plains; therefore between *a* A, B there falls one mean Proportional; seeing therefore the first A is a
b 22 8 Square, also the third B *b* shall be a Square.

3. If two Numbers (A and B) produce a Number that is not a Square, although A be a Square, yet B shall not be a Square.

For if B were also a Square, then the two A and B, would produce a Square Number by *Corol.* 1. contrary to the Hypothesis.

4. The Square A, and B which is not a Square, produce a Number that is not a Square, otherways by

Cor.

Cor. 2. B would alſo be a Square, contrary to the Hypotheſis.

PROP. III.

A Cube Number (CCC) being multiplied into its ſelf, produces a Cube D.

Let the Root or Side of the Cube be C, then by *Cor.* to *Prop.* 23. *Eu.* 8. 1, C, CC, CCC, are in continual Proportion; and therefore between 1, and CCC, there falls two mean Proportionals, but becauſe CCC x CCC = D, it will follow *a* 1 .. CCC :: CCC .. D; therefore alſo between *b* CCC and D there falls two means, wherefore ſeeing the firſt CCC is a Cube, alſo the fourth D will be a *c* Cube, which was to be Demonſtrated.

CCC		
CC		
C		*a Def.* 13
D	1	
		b Prop. 8 8
		c 23 8

PROP. IV.

If a Cube A be multiplied by a Cube B, the product AB, ſhall be a Cube.

A x A = AA and *a* AA is a Cube; and becauſe A and B are Cubes, there falls two *b* mean Proportionals between them; but AA .. AB :: *c* A .. B. therefore alſo between AA and AB there *d* falls two means; wherefore becauſe the firſt A is a Cube, the *e* fourth AB. alſo ſhall be a Cube, W. W. Demonſtrated.

A.	B.	*a Prac.*
		b 12 8
AA	AB	*c Sch.* 17 7
		d 8 8
		e 23 8

PROP. V.

If a Cube A, multiplying any other Number B, generates a Cube AB; alſo the Number B multiplied ſhall be a Cube.

Cube A x A = AA, but AA *a* will be a Cube; *a* 3 9

A. B
AA. BB

b 12 8
c Sch. 17 7
d 8 8
e 23 8

now becauſe AA and AB are both Cubes, there will fall two *b* mean Proportionals between them, but AA .. AB :: *c* A .. B; therefore alſo between A *d* and B there falls two means; wherefore becauſe the firſt A *e* is a Cube, the fourth B ſhall alſo be a Cube. W. W. Demonſtrated.

Corollary.

1. OF a Cube (A) multiplied by a Number X that is not a Cube, is made a Number that is not a Cube; for otherwiſe by Prop. 5. alſo X would be a Cube, contrary to the Hypotheſis.

2. If a Cube A multiplied by B maketh a Number that is not a Cube, neither ſhall B be a Cube; otherwiſe by Prop. 4. alſo the Fact of A x B would be a Cube, contrary to the Hypotheſis.

PROP. VI.

IF a Number A multiplying its ſelf, produces a Cube B, that Number A its ſelf is a Cube.

A x B = E, becauſe A x A = B, and A x B = E, it is manifeſt that E is *a* a Cube; and becauſe that A x B Cube, = E Cube, alſo the Cube B x A = *b* E Cube, therefore alſo *c* A is a Cube, which was to be Demonſtrated.

a Def. 29
b 17. 7
c 5. 9

A.
B. E

PROP. VII.

IF a Compoſit Number A multiply any other Number B, the Number produced E ſhall be a Solid Number.

Some

Some Number (beſides a Unit) meaſure a Compoſit Number (which let be C) by ſome other Number, which let be D; *a* therefore C x D = *b* A but A x B = *c* E; therefore E is made from the Multiplication of the three Numbers C. D. B. therefore E is a *d* Solid Number, which was to be Demonſtrated.

A. B. E
C. D

a Def. 22
b Ax. 8
c Hyp.
d Def. 26

Lemma.

IN a Series of Numbers, continually Proportional and beginning at Unity (1, A, B, C, D, E, *&c.*) *The firſt Number A from Unity, multiplying any of the other D, produces the next following Number E.*

For it will be *f* 1 . . A : : D . . E; *from whence this is manifeſt.*

f Def. 13

PROP. VIII.

IF from a Unite there be Numbers how many ſoever, continually Proportional, the ſecond Number B (Unity not being reckoned) ſhall be a Square, and ſo all following D, F, N, &c. leaving one between.

But the third C is a Cube, and ſo all the following F, K, *&c.* intermitting two.

The ſixth F, is both a Cube Number, and a Square and ſo all the following, intermitting five;

1	2	3	4	5	6	7	8	9	10	11	12	13
1.	A,	B,	C,	D,	E,	F,	G,	H,	K,	L,	M,	N,	O.
1.	2.	4.	8.	16.	32.	64.	128.						

Part. 1. Becauſe 1 . . A : : A . . B it is manifeſt that A x A = *a* B, and conſequently *o* B is a Square. Alſo becauſe B, C, D are continual Proportionals; and that B is a Square, alſo *b* D ſhall be a Square, and ſo on intermitting one.

a Def. 13
o Def. 28
b 22 8

Part 2.

c Lem. d Def. 29 e 23 8

Part. 2. Becauſe A x A = B, and again c A x B = C, therefore d C ſhall be a Cube: Laſtly becauſe C, D, E, F, are four continual Proportionals, and that C is a Cube, alſo F ſhall be a e Cube; and ſo on, intermitting always two.

Part. 3. This is manifeſt from Part 1 and 2.

SCHOLIUM.

Because the firſt Number in a Series of continual Proportionals, multiplying any other in the ſaid Series, produces the next following; it is manifeſt that a Series of continual Proportionals, may excellently be expreſt, only by the continual ſetting together of the Letter A, repreſenting or deſigning any Number, after this manner.

•	1	2	3	4	5	6
1.	A.	AA.	AAA.	AAAA.	AAAAA.	AAAAAA.
A.	7.	A. 8.	A. 9.	A. 10.	A 10.	

That is A x A = AA. a Square; A x AA = AAA a Cube; and ſo on, whence the whole Demonſtration of the Propoſitions is ſeen by once looking on; for becauſe the Product of Multiplication is ſhewn by the putting together of Letters, it is manifeſt that the Product will be a Square, when the number of Letters is even, which conſequently a Binary will meaſure, as is done in the ſecond, third, and fourth place, and in all the following, intermitting one.

Again it is manifeſt that the Product is a Cube, when Three or a Ternary meaſures the Number of Letters, and ſo may be Triſected, which happens in the third and ſixth place, and in all the following, intermitting always two: Laſtly it is manifeſt, that when the multitude of Letters is a prime Number, as 5, 7, 11, 13, *&c.* that it hath neither a Cube nor a Square; for ſeeing no Number meaſures prime Numbers, the Number of Letters cannot be Biſected,

ed, as is required to a Quadratick, nor Triſected as to a Cubical multiplication: Theſe Numbers are uſually by Arithmeticians called Surdeſolids or Superſolids, but all the other terms of the Progreſſion are Squares or Cubes, or both; becauſe that ſeeing the Numbers of Letters or Dimenſions, whereof they conſiſt, are compoſit Numbers, either a Binary or Ternary, or both together, meaſure them, and conſequently they may be either Biſected or Triſected, or both.

Laſtly, The Number of Letters in the Terms of the Progreſſion already ſpoken of are called Poteſtates, and are wrote over them in a Natural order, which ſhew the place or Dimenſions of each, and are alſo called Exponents; ſo in the foregoing Series, which we uſed before the Scholium: A is the firſt Term, and is called the Root; B is the Second or of two Dimenſions, and is called a Square, C the third or of three Dimenſions, is called a Cube, *&c.* In the other Series the Number of Letters, expreſs the Place, the Sides, and Dimenſions of each power.

From what is above ſaid it is manifeſt, 1. That in every Series of continual Proportionals, the Term whoſe exponent is 6, is a Square and a Cube together, for the Number of its Sides, ſeeing it can be meaſured by 2 and 3, may be Biſected and Triſected. 2. That intermitting always 5, all the reſt are together Cubes and Squares, to wit the 12, 18, 24, Term, and the reſt whoſe Exponent's a Senary meaſures; for ſeeing both 2 and 3 meaſures 6, and that 6 meaſures 12, 18, 24, *&c.* alſo as well 2 as 3 ſhall meaſure the ſame, which conſequently may be Biſected and Triſected; therefore, *&c.*

PROP. IX.

IF in a Series of Numbers continually Proportional from a Unite, the firſt Number A be a Square, all the reſt ſhall be alſo Squares.

If

If the first is a Cube, all the rest shall also be Cubes.

Part 1. Because A by Hypothesis, and B by the preceeding are Squares, and that A x B = C also *a* C shall be a Square; again because □A x □C = *b* D also D shall be a Square, and so on.

a Cor. 1, 2. 9.
b Lemn.

Part 2. Because A by Hypothesis is a Cube, and that A x A = *B*; also B shall be a *c* Cube, and because the Cube A x Cube B = *d* C, also C shall be a *e* Cube, *&c.*

c 3. 9.
d Lemn.
e 4. 9.

PROP. X.

IF of Numbers continually Proportional from a Unite, the first A be not a Square; neither shall any other be a Square, but the second B, and the rest following D, F. intermitting always one Term.

And if the first is not a Cube; neither shall any other be a Cube, but the third, and the rest following F *&c. intermitting two always.*

1. A. B. C. D. E. F. G.

Part 1. If it can be, let E be a Square Number; therefore because E .. D :: B .. A, and that *a* E *b* D *b* B are Squares, also A shall be a *c* Square, contrary to the Hypothesis.

a Hyp.
b 18. 9.
c 24. 8.

Part 2. If it be possible let D. be a Cube, seeing therefore *d* F and C are also Cubes, and that by Hypothesis and equallity of Reason it is F. : C :: D. . A, also *e* A shall be a Cube, contrary to the Hypothesis.

d 8. 9.
e 25. 8.

PROP. XI.

IN a Series of Numbers continually Proportional from Unity, any of the lesser Terms C. measureth the greater G. by some Numbers which is in the Series.

1. *A. B; C. D. E. F. G.* for by equality of Reason, C .. G :: 1 .. D, therefore C measures a G by D.

a Def. 19.

Corol-

Corollaries.

1. HOW far the greater Number G is distant from the Number C which measures it, so far is D the Number by which the lesser Measures the greater distant from Unity.

2. The first Number A measures any other Number D. by that which goes before it C.

3. Any Number C in a Series, multiplying its self, produces a Number F so far distant from its self, as its self is distant from Unity.

4. If any Number B. in a Series multiplyeth any other Number D in that Series, as far as Unity is distant from the lesser B, so far the greater D shall be distant from the Product F.

PROP. XII.

IF from a Unite Numbers how many soever are continually Proportional, the prime Number (E) that measures the last D, shall also measure A that is next to the Unite. 1. A. B. C. D. E. H. G. F.

For, if it be possible, let the prime E, not measure A, therefore *a* E and A are Primes, the one to the other, but because E measures D, let it be by F, and A measures the same *b* D by C, therefore E.. A :: *c* C.. F; but because E and A are Prime the one to the other, they are *d* the least in their Proportion; therefore E. measures C, *e* suppose by G.

Wherefore seeing *f* A measureth the same C by B, it will again be E.. A :: *g* B.. G. therefore because E, A, are the least in their Proportion: E again measures *h* B suppose by H; but also A measures B by A (for A x A = D, as is manifest from the 8) therefore again *i* E.. A :: A.. H; wherefore seeing E, A. are the least in their Proportion E *k* measureth A, which was to be Demonstrated.

a 31. 7.
b Cor. 2. Præced.
c Cor. 19. 7.
d 23. 7.
e 21. 7.
f Cor. 2. Præc.
g Cor. 19. 7.
h 21. 7.
i Cor. 19. 7.
k 21. 7.

PROP.

PROP. XIII.

IF from a Unite be Numbers continually Proportional, and the next to the Unite A be a Prime Number, then no other Number ſhall meaſure the greateſt D, but thoſe which are among the Proportional Numbers.

1. For, if it be poſſible, let another E different from the ſaid A, B, C meaſure the greater D. but E muſt not be a Prime; otherways E would alſo meaſure *a* A, and conſequently A would not be a Prime contrary to the Hypotheſis; therefore E is compoſit, and ſo *b* ſome Number meaſures it.

1. A. B. C. D.
E. F. G. H.
O.

a Præced.

b 24. 7.

2. Which I ſay is A, for if any other Prime O, meaſure E, then becauſe E meaſures D; alſo O would *c* meaſure D, therefore alſo the *d* ſaid A, which is abſurd, ſeeing A is a Prime: Now let E meaſure D by H.

c Ax. 11.
d Præced.

3. H, ſhall be different from A, B, C, for if it be poſſible let H be the ſame with any of the ſaid A, B, C; ſuppoſe C, therefore E meaſures D by C, and conſequently C alſo *e* meaſureth D by E, therefore E is one *f* of the Rank A, B, C, which is abſurd; ſeeing you would have E not to be one of the Rank A, B, C, therefore H is different from the ſaid A, B, C.

e Ax. 9.
f 11. 9.

4. Alſo H will not be a Prime Number; otherways ſeeing H meaſureth D, it would alſo *g* meaſure A. which is Prime by the Hypotheſis, which is abſurd; Therefore H is a compoſit Number.

g Præced.

5. But becauſe H is a compoſit Number, ſome Prime Number will meaſure it, which I ſay is A, for if any other Prime O ſhould meaſure H, ſeeing H meaſureth D (for becauſe E meaſures D by H, alſo H ſhall *h* meaſure D by E) alſo *i* O ſhall meaſure the ſaid

h Ax. 9.
i Ax. 11.

ſaid D, and conſequently alſo the ſaid *k* A a Prime which is abſurd. *k Præc.*

6. Now becauſe A meaſureth *l* D by C, and that alſo E meaſureth D by H, it will be A . . E : : *m* H . . C, wherefore ſeeing (as I ſhew in Number 1) A meaſures E, alſo H ſhall meaſure C, ſuppoſe by G; becauſe therefore, as E, being divers from A, B, C, did meaſure the laſt D by H; ſo now H which I have already ſhewn in Numb. 3. to be diverſe from A, B, C, meaſures the laſt C of the ſaid A, B, C by G; after the ſame manner I will ſhew that G is diſtinct from A, B, and that it is not a Prime; and that it may be meaſured by A; which three I have ſhewn of the Numb. H. *l Cor.* 2. 11. 9. *m Cor.* 19. 7.

Therefore becauſe A meaſureth C *n* by B, and H meaſureth C by G, it will be *o* A . . H : : G . . B. but I have ſhewn Number 5. that A meaſures H, therefore alſo G meaſures B, ſuppoſe by F. I may again ſhew that F is different from A, as I have plainly ſhewn in Numb. 3. that H is different from A, B, C, and G from A, B. Wherefore becauſe A meaſures B by A, and G meaſures B by F, it will be A . . G : : *p* F . . A; but it is ſhewn Numb. 6. that A meaſures G; therefore alſo F meaſures A, which is abſurd, ſeeing A is a Prime Number; But this abſurdity is deduced from this, that E is put different from A, B, C, and to meaſure D; therefore it is manifeſt that no Number diſtinct from A, B, C meaſures D, which was to be Demonſtrated. *n Cor.* 2, 11, 9. *o Cor.* 19. 9. *p Cor.* 19. 7.

PROP. XIV.

THE leaſt Number A which certain Prime Numbers given B, C, D do meaſure, can be meaſured by no others than thoſe given.

A. B. C. D E. F

If it be poſſible let ſome other Prime E meaſure the leaſt A by F therefore E x F = *a* A and conſequently E, F. are the Sides of the ſaid A, becauſe therefore B, C, D meaſure A, they ſhall alſo *b* meaſure one of the ſaid E, F, not E, the Prime; therefore F: but F is leſs then A; for F x E = A; therefore A is not the leaſt which B, C, D meaſure, which is againſt the Hypotheſis.

a Ax. 8

b 32. 7

PROP. XV.

IF three Numbers AA, AB, BB, continually Proportional, be the leaſt in their Proportion, any two added together ſhall be Prime to that which is left.

A. B AA. AB. BB

From Prop. 2, 8. it is manifeſt that, if A, B are ſuppoſed to be the leaſt in the Rario given, that then AA. AB. BB ſhall be the three leaſt in the ſame Proportion: Now becauſe A and B are *a* Prime the one to the other; *b* alſo A + B is Prime to B, but alſo A is Prime to B, therefore alſo *c* A + B x A (= AA + AB) is Prime to B; wherefore AA + AB ſhall be alſo *d* Prime to BB; after the ſame manner may be ſhewn that BB + AB is Prime to AA.

a 24. 7
b 30. 7
c 26. 7
d 27. 7

It remaineth to ſhew that AA + BB is Prime to AB, becauſe A, B are Prime the one to the other; and ſo alſo *e* as well A as B is Prime to A + B: alſo AB = A x B ſhall be *f* Prime to A + B; therefore AB is *g* Prime to the Square of the ſaid A + B that is to *h* AA + BB + AB, therefore by dividing, AB is Prime to *i* AA + BB + AB; and by *k* dividing again, AB is Prime to AA + BB. W. W. Demonſtrated.

e 30. 7
f 26. 7
g 4. 2
h 27. 7
i 30. 7
k *ibid.*

PROP. XVI.

TO two Numbers A and B that are Prime the one to the other, there cannot be given a third Number in continual Proportion with them.

If

For if it be poſſible, let it be A . . B : : b . . C becauſe A and B are Prime the one to the other, they are the *a* leaſt in their Proportion; therefore A *b* meaſures b; but alſo A meaſures its ſelf; therefore A and b are *c* compoſed the one to the other, contrary to the Hypotheſis.

A.	B.	b.	C.
5	7	7	

a 23. 7.
b 21. 7.
c Def. 24.

PROP. XVII.

IF in a Series of Numbers continually Proportional, the Extremes A, D are Prime the one to the other; the ſaid Series cannot be continued further.

For if it be poſſible let it be A . . B : B . . C, *&c.* ſo D to ſome other E; therefore by Permutation A . . D : : B . . E, but A and D are *a* Prime one to the other, and conſequently the *b* leaſt in their Proportion; therefore *c* A meaſures B, but B meaſures *d* C, and C meaſures D; therefore A *e* meaſures D, but A alſo meaſures it ſelf; therefore *f* A and D are compoſed the one to the other, which is againſt the Hypotheſis.

A.	B.	C.	D.	E

a Hyp.
b 23. 7.
c 21. 7.
d 1. 9.
e Ax. 11.
f Def. 24.

PROP. XVIII.

TWO Numbers A. B. being given to conſider, whether there can be found a third Proportional to them.

If A, B, are Prime the one to the other, there cannot be found a third which is manifeſt from the 16. Prop.

A.	B.	X.
	BB.	

If A, B, are not Prime the one to the other; B x B = BB, if A meaſures BB, ſuppoſe by X, then X ſhall B a third Proportional, for X x A = *a* BB; and alſo B x B = BB; therefore *b* A . . B : : B . . X.

a Ax. 8.
b 20. 7.

A. B. Z. BB. If A doth not measure BB, a third Proportional cannot be had.

If it possible, let Z be it; therefore A x Z = BB = *c* B x B therefore because A x Z = BB, A shall measure *d* BB by Z, contrary to the Hypothesis.

c the same
d Ax. 7.

PROP. XIX.

THREE Numbers being given A, B, C, to consider whether there can be found a fourth Proportional to them.

A. B. C. X BC. The second B x third C = BC, if the first A measures BC, suppose by X; then X will be a fourth Proportional, for then A x *a* X = BC as well as B x C = BC; therefore *b* A . . B : : C . . X.

If A does not measure BC, a fourth Proportional cannot be gotten; if it can let Z be the Number, then A x Z = *c* BC = B x C; therefore A *d* measures BC by Z. contrary to the Hypothesis.

a Ax. 8.
b 19. 7.
c 19. 7.
d Ax. 7

SCHOLIUM.

THE 4 preceding Propositions are to be understood of whole Numbers; for to any two Numbers, a third, and to any three a fourth Proportional may be found in Fractions, as shall be taught in Practical Arithmetick.

But this is worthy Observation, which we will Demonstrate in the 7 Chap. Book 5. of Practical Arithmetick; *that although to any two a third Proportional, and to three a fourth may be found, at least in a Fraction, yet between those that an Integral mean cannot be found, neither can a Fractional mean be found.*

A. B. C. Z D. *Three Numbers A, B, C, being given a fourth Proportional is also found in this manner.*

The

The first A dividing the second B maketh the Quotient D, the third C multiplying D generates Z, which is that sought.

For seeing A divides B by D, also a $A \times D = B$, *but* a Ax. 8
also $C \times D =$ b Z, *therefore* $A .. B :: c C .. Z$. b 17. 7
It will be impossible to find a fourth Proportional c 7. 7
which is an Integer (not if A does not measure B but) if the fact of C and D be not an Integer, or reducible to an Integer.

PROP. XX.

PRime Numbers are infinite, or there are infinite Number of Prime Numbers a. a 38. 7

Let there be given three Prime Numbers A. B. C. find the least X, which A, B, C, measure; to which add one, if then X + 1 makes a Prime Number, you will have a fourth Prime: If X + 1 does not make a Prime, some *b* Prime Number will measure it, suppose Z; this will be a Prime different from the given A, B, C, for if it be possible let it be the same with one of the given Primes C; therefore because C measures X, also Z will measure X, but also Z measures X + 1. Therefore Z will measure *c* 1, also which is absurd; therefore Z is a new Prime; after the same manner may be found more, infinitely.

A. B. C
X + 1
Z
b 34. 7
c Ax. 12

PROP. XXI.

EVen Numbers how many soever A, B, C, being added together, make an even Number.

Because A, B, C are even Numbers, let their *a* halves be N, O, P. Therefore because $A .. N :: B .. O :: C .. P$ $.. A + B + C .. N + O + P$, but A is the double of N; therefore A + B + C is also the double of N + O + P. and

A. B. C
N. O. P
a Def. 15
b 12. 5

consequently N † O † P is half of A † B † C; there-
c Def. 16 fore A † B † C is an *c* even Number, which was to be Demonstrated.

PROP. XXII.

Odd Numbers A, B, C, D, even in Number, compose an even Number.

Take a Unite from each, then what
a Def. 17 A, B, C, D remains shall be *a* even Numbers, and the Unites taken away will compose or
b Hyp. make an *b* even Number; wherefore by the preceding the Numbers left and those taken away together (that is A, B, C, D) compose an even Number.

PROP. XXIII.

ODD Numbers A, B, C, odd in Number, compose an odd Number.

Because an odd Number differs from
A. B. C. an even Number by a Unite, C—1 *c*
c Def. 17. C.—1 shall be an even Number; but the odd
d Prac. Numbers A, B *d* compose an even Number; therefore A, B, C—1. compose or make an even Number, but the summ of A, B, C —1. differs from the Sum of A, B, C by a Unite; therefore
e Def. 17. the Sum of A, B, C is an *e* odd Number which W. Demonstrated.

Corollary.

IN like manner an odd being added to an even Number make an odd Number.

PROP. XXIV.

IF from an even Number D be Substracted an even Number E, the remainer F, shall also be an even Number.

For

For if the remainer F were odd, with the even Number E, it would compoſe an *a* odd Number; but F and E compoſe D; therefore D would be odd, contrary to the Hypotheſis.

D
E
F

a Cor. Præ.

PROP. XXV.

IF from an even Number A, be Subſtracted an odd Number B, the remainer C will be odd.

For if the remainer C were even, that with the odd Number B would make an *b* odd Number; but C with B makes A; therefore A would be odd, contrary to the Hypotheſis.

A
B
C

b Cor. 23. 9

PROP. XXVI.

IF from an odd Number E, be Subſtracted an odd Number F, the remainer G will be even.

For if G were odd, that with F would make E an *c* even Number, contrary to the Hypotheſis.

G
E
F
G

c 22. 9.

PROP. XXVII.

IF from an odd Number A, be Subſtracted an even Number B, the remainer C will be odd.

For if C were even, it with the even Number B, would compoſe an even Number *a* A, contrary to the Hypotheſis.

A
B
C

a 21. 9

PROP. XXVIII.

AN *even and an odd Number A, B, being multiplied together, produce an even Number C.*

b Def. 13 For C is *b* Compoſed of an even Number A, ſo often taken as there are Unites in B; therefore C is an *c* even Number.

A 2 B 3
C 6

c 12. 9

Corollaries.

1. AN even Number multiplying an even Number, generates an even Number; this is manifeſt from *Prop.* 21.

2. Every evenly odd Number is even; this is manifeſt from *Def.* 19, *Ax.* 7. and this *Prop.*

PROP. XXIX.

AN *odd Number D multiplying an odd Number E, produces an odd Number F.*

a Def. 13 For F is *a* compoſed of D, ſo often taken as there are Unites in the odd Number E, therefore F is *b* odd.

b 12 9

Corollaries.

1. IF *an odd Number A, meaſures an even Number B, it meaſures the ſame by an even Number C.*

Ax. 8
d 29. 9

12 B } C A
3 A }

For if C were odd, becauſe A x C = *c* B, B would be *d* odd (being produced of an odd A, and an odd C which is againſt the Hypotheſis.

2. *An odd Number D meaſuring an odd Number E, meaſures it by an odd Number F.*

For

For if F were even, because D odd and F even, being multiplied, produce *a* E. F would be *b* even, contrary to the Hypothesis.

15 E
3 D } F 5 *a* Ax. 8 *b* 28 9

SCHOLIUM.

From what is already said, it may be demonstrated, that no Number (AB) can be so cut or divided, that the Product of the whole (AB) multiplied into one part (CB) is equal to the Square of the other part AC; and consequently that the 11 Prop. 2 Book Eucl. *cannot be any ways applicable to Numbers, although the* 10 *First* Propositions *of that* Book *are also true in Numbers.*

For if it be possible, let AB x CB = □ AC. *Now either* AC *is even, and* CB *odd, or* AC *is odd, and* CB *even, or both* AC *and* CB *are odd, or both are even.*

A——C——B
D——E——

First, Let AC *be even, and* CB *odd; therefore the whole* AB *is* a *odd; therefore* AB x CB *is* b *odd, but the Square of* AC *even, is* c *even, therefore the odd Number produced of* AB x CB, *is equal to the even Number, to wit, the* □ *of* AC, *which is absurd.*

a Cor. 23 9
b 29 9
c Cor. 28 9

Secondly, Let AC *be odd, and* CB *even, therefore the whole* AB *is* d *even; therefore* AB x CB *is* e *even, but the Square of the odd* AC *is* f *odd; therefore again this odd Number is equal to that even Number, which is absurd.*

d Cor. 23. 9
e 28 9
f 29 9

Thirdly, Let both AC *and* CB *be odd, therefore the whole* AB *is* g *even, therefore the fact of* AB *even, in the odd* CB *is* h *even; but the Square of the odd* AC i *is odd, therefore again this odd Number is equal to that even number made of* AB x CB, *which is absurd.*

g 22 9
h 28 9
i 29 9

Lastly, let both AC, CB, *be even. Take two numbers,* D, E, *the least in the reason of* AC *to* CB, *then* D, E *are* k *prime the one to the other, and consequently* [illegible] k 24 9

cannot be even, otherways a Binary would measure them, but either both are odd; or D is even and E odd; or D is odd and E even.

Now because AB x CB = □AC *it will be* AB . . AC :: AC . . CB, *and because* E . . D :: BC . . CA *it will be by Composition* $\overline{E + D}$ . . D :: BA . . AC :: *l* AC . . CB :: *m* D . . E. *Now because* D + E . . D :: D . . E, *therefore the fact of* D + E x E = □ D, *which that it cannot be is Demonstrated in the three first parts.*

l shewn before.
m Hyp.

But some perhaps will think that a Number may be cut or divided into two such parts as have Fractions belonging to them; but if those parts are reduced to two Fractions of the same Denomination, as shall be taught in Pract. Arith. Book. 2 Ch. 3. Prob. 2. *it will easily appear that that also is impossible.*

PROP. XXX.

IF an odd Number B measures an even Number A, it also measures its half.

Let the odd B measure the even A by C, then C is *a* even, and C shall also measure A *b* by B; therefore also the half of the said C shall measure the half of A by *B* (for C . . A :: $\frac{1}{2}$C . . $\frac{1}{2}$A) therefore also B measures the *c* half of A by the half of C which was to be Demonstrated.

12 A, 3 B } C 4.

a Cor. 1. 29. 9.
b Ax. 9.
c Ax. 9.

PROP. XXXI.

IF an odd Number A be Prime to any Number B, it shall also be Prime to C the double thereof.

For if A and C are not Prime to one another, let some Number D measure them, A by E, then D will of necessity be odd, otherways D even multiplyed by E would produce A *a* even; wherefore because D odd measures C even (for C is even, seeing it is double of B) also D shall measure *b* B the half thereof; but you suppose D measures A; therefore

A. 3. B. 5. C 10. D, E

a 28. 9.
b Prac.

fore A, B, are not Prime to one another, contrary to the Hypotheſis.

Corollary.

AN odd Number A, which is Prime to any Number B is alſo Prime not only to the double of B, but alſo to its Quadruple and Octuple, *&c.* which is manifeſt from this Prop.

PROP. XXXII.

IF a double Proportion be continued from a Unite infinitely, it will only give all the evenly even Numbers.

That they are all evenly even is manifeſt from Prop. 11. Book 9. For by that meaſures any in the Series, by one of the Series, that is, by an even number; therefore by *Def.* 18.

1.	A,	B,	C,	D,	E,	F
1.	2.	4.	8.	16.	32.	64
	1,	R,	P,	O		

That they are only evenly even is manifeſt from Prop. 13. Book 9. for ſeeing 2 is the next Prime to a Unite, no Number will meaſure any of them 2, 4, 8, *&c.* beſides them which are in the Series, that is, beſides the even Number; therefore by *Def.* 18.

That no Number beſides theſe are evenly even only I thus ſhew; let there be only any evenly even Number O, whoſe half is P, this will be even, otherways if P were odd, a Binary would meaſure O by the odd Number P, ſo O would be evenly odd contrary to the Hypotheſis; therefore let R be the half of the ſaid P; this again will be even, for if R were odd, ſeeing it is the fourth part of the ſaid O, the Number 4 will meaſure O by the odd Number R contrary again to the Hypotheſis; and ſo always the half of the preceding will be even, untill we come to Unity; therefore let 1 be the half of R, then R. P. O ſhall be doubled from Unity, and conſequently

O ſhall be one of the Series, therefore the propoſed is manifeſt.

PROP. XXXIII.

IF *all the odd Numbers be doubled, it will but produce all the evenly odd Numbers only* (4. 6. 10, *&c.*)

a Hyp. They are all evenly odd, becauſe their half is *a* odd, therefore a Binary ſhall meaſure every one by a half odd Number.

3. 5. 7. 9. 11.
6. 10. 14. 18. 22.
A. B. C. D. E.
7 N 2
P Q

That they are only evenly odd, I thus ſhew; Take any of them, ſuppoſe D, whoſe half let be N, and ſet a Binary under it. If therefore D is alſo evenly even, *b* P even meaſures it by Q even; therefore becauſe N alſo meaſures the ſame D by 2. it will be *c* Q. . N :: 2 . . P, but 2 meaſures the even Number P. therefore alſo Q even, meaſures N odd, which is abſurd, and contrary to *Prop.* 21. *E.* 9.

b Def. 18.

c Cor. 19. 7

That none beſides theſe are evenly odd only, I thus ſhew, if there are, they muſt be *d* even; but even Numbers either have even halves, and ſo are doubled from Unity, and therefore by the preceeding, they are only evenly even, or they have odd Halves, and ſo by the following they are evenly even, and evenly odd; therefore, *&c.*

d Cor. 28. 9

PROP. XXXIV.

EVen *Numbers* (A, B, C, D, E) *which are neither doubled from two, nor have their half parts odd, are evenly even, and evenly odd, and beſides there are no other.*

They are all evenly even, becauſe that ſeeing their

Halves

Halves are even, a Binary measures them by those even Halves; therefore by *Definition* 18, *&c.* And that they are also evenly odd, I thus shew: Take any of them as B, which if you divide into half, and the half again into half, and so on, at length you will happen on some odd Number (if we should always happen on even Numbers, at length we should come to 2, and so B would be doubled from Unity, contrary to the Hypothesis) which odd Number will measure the even Number B, by an *b* even Number; therefore B is evenly odd.

36,	40,	44,	48
E.	F.	G.	H.
12,	20,	24,	28
A.	B.	C.	D.

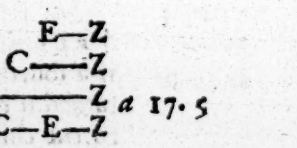
b Cor. 1. 29

Also from hence is manifest, the third part of the preceding *Prop.* And that besides these, there are no other evenly even, and evenly odd Numbers, is manifest from the two preceeding Propositions.

PROP. XXXV.

IF *there are Quantities, how many soever* (EZ, CZ, BZ, AZ) *in continual Proportion, then as* CE, *the excess of the second* (CZ) *above the first, or least* EZ *is to the least* (EZ) *so is the excess* (AE) *of the greatest* (AZ) *above the least* (*to all the rest*) BZ, CZ, EZ, *taken together; that is,* CZ — EZ = CE .. EZ :: AZ — EZ = AE .. BZ + CZ + EZ.

Transfer BZ, CZ, EZ, into the greatest AZ, because AZ .. BZ :: BZ .. CZ :: CZ .. EZ, therefore by *a* Division, AZ — BZ .. BZ :: BZ — CZ .. CZ :: CZ — EZ .. EZ, that is, AB .. BZ :: EC .. CZ :: CE .. EZ. Wherefore as *b* AB .. BZ, or CE .. EZ :: AB + BC + CE .. BZ + CZ + EZ, which was to be Demonstrated.



a 17. 5

b 12 5.

From

From this Propoſition, whoſe Fœcundity former Arithmeticians ſeem not to have obſerved, we have deduced the following

Corollaries.

1. IF there are Numbers how many ſoever, EZ, CZ, BZ, AZ, continually Proportional, then it will be, as the Denominator of the Proportion, or Ratio, leſs a Unit is to a Unit, ſo is the difference AE of the leaſt and greateſt terms, to the ſum of all the terms leſs the greateſt AZ.

Let FN. repreſent the Denominator, and GN the Unit; therefore FN . . GN :: CZ . . EZ, and by Diviſion FN — GN . . GN : : CZ — EZ . . FZ, that is, FG . . GN : : CE . . FZ; but by this Prop. CE . . EZ :: AE . . BZ † CZ † EZ; therefore alſo FG . . GN : : AE . . BZ † CZ † EZ.

2. The ſame being poſited, if the Denominator FN, and difference AE of the greateſt and leaſt Terms be given; the ſum of all the Terms, except the greateſt will be had, if the difference AE of the greateſt and leaſt be divided by FG, the Denominator leſs a Unit.

For ſeeing *c* FG . . GN a Unit : : AE . . BZ † CZ † EZ = ſum of all the Terms — AZ, it is exhibited if a fourth Proportional be found *d* and that is obtained, if the ſecond Term, to wit, Unity be drawn into the third, and the Product, which is ſtill AE be divided by the firſt FG.

c Cor. 1 Præc.

d 19

3. But if the Proportional Terms are Magnitudes, the ſum of all leſs, the greateſt will be had, if as the difference CE of two collateral Terms, is to the leſſer EZ, ſo is AE the difference of the greateſt and leaſt Terms to another; this is manifeſt from the Prop.

4. If

4. If the given Series of proportionals whether Numbers or Magnitudes be in duple Proportion, the Summ of all the terms except the laſt will be had, if the leaſt be taken from the greateſt.

For by this 35 Prop. CE .. EZ :: AE .. BZ † CZ † EZ; but here CE = EZ therefore alſo AE = BZ † CZ † EZ.

5. The ſame being poſited, as the difference of the two greateſt terms is to the greateſt term, ſo is the greateſt leſs the leaſt to the Sum of all the terms leſs the leaſt, that is, AB = AZ — BZ .. AZ :: AE = AZ — EZ = S — EZ.

It is ſhewn in this Prop. AB .. BZ :: AE .. BZ † CZ † EZ; therefore AB .. AZ :: AE .. AE † BZ † CZ † EZ, that is, all except the leaſt EZ.

SCHOLIUM.

HE that compares this Propoſition and its Corollaries, with what I have wrote in Prop. 11. Eucl. 6. *will eaſily underſtand that from hence is deduced all that contemplation; whereby in a Series of Infinite Proportional quantities one is exhibited equal to all the Terms; for when the Number of Proportionals is finite, then* f *CE .. EZ; or AB .. BZ :: AE (that is the greateſt leſs, the leaſt or laſt) .. BZ † CZ † EZ; but when the Number of Proportionals is infinite, then AB .. BZ :: greateſt AZ whole .. BZ † CZ † EZ, &c. where there is only this difference, that in the third place all the greateſt Number AZ is taken, becauſe the leaſt which before was Subſtracted from the greateſt is now nothing, and ſo cannot be Subſtracted.*

f *By this 35. Prop.*

Again when the Number of Proportionals is finite, AB .. g AZ :: AE .. AE † BZ † CZ † EZ. But when the Number of Proportionals is infinite, then AB .. AZ :: AZ .. AZ † BZ † CZ † EZ; where again, this is only the difference, becauſe the leaſt does now vaniſh and is nothing, and ſo cannot be Subſtracted.

g *Cor. 5. Præc.*

PROP.

PROP. XXVI.

IF a Series of Number: continually Proportional from Unity in a duple Ratio (A, B, C, D) be continued until their Sum be a Prime Number E, the Sum being Multiplyed into the greateſt Term D ſhall produce a perfect Number.

Let there be taken ſo many Numbers E, O, N, P double from E as are the Numbers
a 22. 5. 1. A, B, C, D. E, O, N, P, F. M. Q. A, B, C, D by *a* equality; therefore A .. D :: E .. P; wherefore A x P = *b* D x E = F; therefore P meaſures F by A a Binary, and conſequently E, O, N, P. are continually Proportional induple
b 19. 7. *c* Cor. 4. Ratio of the foregoing; therefore F — E = *c* E + O + N + P; but E = *d* 1 + A + B + C + D; therefore F — E = 1 + A + B + C + D + O + N + P. to each add E, then F = 1 + A + B + C + D + E + O + N + P, but the ſaid Numbers are Aliquot parts of the ſaid F, for ſeeing E, O, N, P, F, are continually Proportional, each meaſures
d Hyp. *e* 11 9. *e* F, and for the ſame cauſe every one of 1. A, B, C,
f Hyp. meaſures D, but D meaſures F, for D x *f* E = F;
g Ax. 12. therefore alſo every one of 1. A, B, C, D *g* meaſures F.

It remaineth to ſhew that there is no other aliquot part of the number F. Let M be any aliquot part of the ſaid F, I will ſhew that it is the ſame with ſome of the aforeſaid, A, B, C, D, E, O, N, P; by the ſame reaſon it is denyed to be; for ſuppoſe M not to be the ſame with any of the ſaid A, B, &c. Then becauſe M meaſures F let it meaſure it by
h Ax. 8 Q therefore M x Q = *h* F; but alſo E x D = F; therefore E .. Q :: M .. D, but becauſe B next to Unity
k Hyp. is a Prime, to wit, *k* 2, and M is ſuppoſed to be diverſe
l 13. 9 from all A, B, C; *l* M does not meaſure D, therefore
m Hyp. neither ſhall E meaſure Q; wherefore ſeeing *m* E is a
n 31. 7. Prime, alſo EQ are *n* Prime one to the other, and
o 23 7. therefore the *o* leaſt in their Proportion. Therefore

E

E measures M, and Q *p* measures D; therefore seeing A is a Prime, Q is one *q* of the said A, B, C; therefore let Q be the same with B, and as many as are B, C, D, let so many be taken E, O, N, doubled from E, therefore by equality B .. D :: E .. N; therefore B x N=D x E=F. But because M measures F by Q; also Q x M=F, wherefore seeing Q x M=F and B x N=F, it follows Q .. B :: N .. M, but I have already shewn that Q is the same with B; therefore N is the same with M. Therefore F hath no other Aliquot parts besides 1. A, B, C, D, E, O, N, P. to all which because F is equal, it is a perfect Number, W. W. Demonstrated.

p 21. 7.
q 13. 9.

SCHOLIUM.

By this Theorem *may be found all the perfect Numbers; because the Sum of* 1, *and* 2, *is* 3 *a Prime Number,* 3 x 2 *makes* 6 *the first perfect Number, whose Aliquot parts are* 1, 2, 3, *and because the Sum of* 1, 2, 4, *is* 7 *a Prime Number,* 7 *multiplied by the greatest* 4 *makes* 28 *the second perfect Number, whose Aliquot parts are* 1, 2. 4, 7, 14; *again because the Sum of* 1, 2, 4, 8, 16 *is* 31 *a Prime,* 31 x 16=496 *the third perfect Number, whose Aliquot parts are,* 1, 2, 4, 8, 16, 31, 62, 124, 248. *The Sum will be easily had, if the following Number be lessened a Unite, as is manifest from* Corollary 4. *preceding, and the Aliquot parts of any perfect Number may be thus found; how many Numbers are taken doubled from Unity secluding Unity, take so many doubled from the Prime, or Sum numbering the Prime; these with those double from Unity, and the Unite its self are the Aliquot parts of the given perfect Number.*

Moreover, the manner of finding the perfect Numbers is briefly exhibited by this Propositioo, and the 4 Preceding Corollary thus; let the Ratio of 1 to 2 be continued infinitely from a Unit; see what Numbers of the Progression by taking away a Unit make or become prime Numbers; for these being drawn into the next preceding them, will give the perfect Numbers.

Of the multitude of perfect Numbers already found, Mar. Mertennus *in his Pref. Sect. 9. writes thus;* That there are but 11 yet found, to wit, 6, 28, 496, 8,128,550, 336, 8589, 869, 056, 137, 438, 691,328, 2 385, 843, 008, 139, 952, 128. *and three other, the last whereof is made of the 257th. Term of a duple Progression, lessened by a Unit, and multiplied into the 256 Term of those* 28, *which* Peter Bungus, *in the* 28 *Chap. of his Book of the* Mysteries of Numbers *rehearses; only the* 8 *first are perfect Numbers (to wit, those delivered above) the others are unperfect Numbers. Thus He.*

But the Reason that more have not been yet found, is because that in a Series of a duple Progression the Intervals of the Numbers, which by lessening by a Unit become Primes, are so very great, for the 11th of the said Numbers is the 257th Term, from Unity exclusive (as is said above) and consequently the Number is very great, and the Labour to find whether or no it be a Prime, extream tedious; if the Number consists of 20 Figures, a whole Age is not sufficient to try it, let one use any way already found.

THE

THE FIRST BOOK OF PRACTICAL Arithmetick.

Of whole Numbers.

IN every Number consisting of many Figures; that Figure which stands furthest towards the Right Hand, is said to be the First, and that which stands furthest towards the Left Hand is the Last.

CHAP. I.

Notation or the Institution of Arithmetical Figures.

THere are 10 Notes, or Characters, (some call them Digits, and some Figures) whereby all Numbers are expresst, to wit,

0. 9. 8. 7. 6. 5. 4. 3. 2. 1.

The first Figure (1) signifies a Unite. (2) signifies two Unites. (3) three Unites, and so on. The last (0) (which is called a Cypher) signifies nothing, but being

being ſet before other Figures augments their value, as ſhall preſently be ſhewn.

Beſides this ſingle value of a Note or Figure already ſpoken of, they have alſo another value, relating to the place every Figure is in, and the value of the places increaſe in a tenfold Proportion continued infinitely.

Therefore any Figure that ſtands in the firſt place ſignifies Unites, any Figure in the ſecond place; ſo many Tens, as it hath Units; in the third place ſo many Hundreds; in the fourth ſo many Thouſands; in the fifth ſo many Ten thouſands; in the ſixth ſo many Hundred thouſands; in the ſeventh ſo many (Thouſand thouſands, or) Ten hundred thouſands, or Millions, as the ſingle Figure hath Unites. After this manner, the value of the following places proceed in a decuple or tenfold Proportion infinitely.

As in the Number A, the firſt Figure (6) ſignifies ſix Units, the Second (9) is valued at nine tens of Units, that is, ninety (90) the third (7) ſeven hundreds 700. or ſeven hundred; the Fourth (5) five thouſands 5000. The fifth (3) three tens of thouſands, or thrice ten thouſands, that is, 30000. thirty thouſand; The Sixth (8) eight hundreds of thouſands, that is 800000. eight hundred thouſand; the Seventh (1) one million or one thouſand thouſands, or ten hundred thouſands; the Eighths value (4) forty millions.

A. 4 1 8 3 5 7 9 6.

9 0. Ninety
7 0 0. Seven hundred
5 0 0 0. Five thouſand
3 0 0 0 0. Thirty thouſand
8 0 0 0 0 0. Eight hundred thouſand

1 0 0 0 0 0 0. a Thouſand Thouſ. or Million.
4 0 0 0 0 0 0 0. Forty Million.

1 Unity
10 Ten
100 a Hundred
1000 a Thousand
10000 Ten thousand
100000 a hundred thousand
1000000 a Million.

Places.	VALUE.
1.	Units
2.	Tens
3.	Hundreds
4.	Thousands
5.	Tens of thousands
6.	Hundreds of thousands
7.	Millions
8.	Tens of millions
9.	Hundreds of millions
10.	Thousands of millions
11.	Ten thousands of millions
12.	Hundred thousands of millions
13.	Millions of millions or bilions
14.	Tens of bilions
15.	Hundreds of bilions
16.	Thousands of bilions
17.	Ten thousands of bilions
18.	Hundred thousands of bilions
19.	Millions of bilions or Trilions
20.	Tens of trilions
21.	Hundreds of trilions
22.	Thousands of trilions
23.	Ten thousands of trilions
24.	Hundred thousands of trilions
&c.	Millions of trilions or quatrilions. &c.

By this Table is ſhewn the value of all the places; which I thought convenient to part thus; that every ſix places may as it were conſtitute ſeveral Members: Therefore in the ſecond Senary are Millions, in the third Bilions, in the fourth Trilions, in the fifth Quatrilions; and ſo on infinitely: For the value of every Senary or Member, is denominated from the number of Senarys or Members going before it; as if you deſire to know the value of the ſeventh Member, it will be Sextilions, becauſe ſix Senarys go before the ſeventh.

If you had rather expreſs the value of the Members by Millions only, repeat the word *Million* ſo often as there are Members preceeding; as if it were required to know the value of the fourth Senary, the word *Million* muſt be ſpoken thrice, becauſe three Senaries preceed the fourth; hence the value of the fourth Senary is Millions of Millions of Millions.

CHAP. II.

Numeration.

TEacheth to write and expreſs any Number given, both which he may eaſily perform, that rightly underſtands both the ſimple and local values of Figures, explained in the foregoing Chapter; but becauſe in great Numbers, many (as well the skillful as others) are ſubject to ſtick; there have been ſeveral ways invented to facilitate this work; of which we will here deliver that which to us ſeems the moſt expedit.

Firſt let the young Arithmetician endeavour readily to write and expreſs the places of the firſt Senary or Member, that is, to write and expreſs Numbers conſiſting of ſix, five, four and three Figures, *&c.*

In

In which he may be helpt by this Praxis.

370, 517. A.
49, 304. B.
7, 059. C.

Let the Numbers A, B, C, be given to be expreſſed; after the firſt three Figures put a Comma, that there may be as if it were two Members, whether the Second conſiſts of three Figures or fewer: Then expreſs the ſecond Member as if it were alone, and to it add the word, *Thouſand*, or *Thouſands* once; but expreſs the firſt Member as it lyes. Therefore you may expreſs the three given Numbers thus;

A. Three hundred ſeventy Thouſand, Five hundred and ſeventeen.

B. Forty Nine Thouſand, Three Hundred and Four.

C. Seven Thouſand, Fifty Nine.

This being premiſed, let any Number, ever ſo great, be given to be numbred, as;

D 96, 638 (908, 003 (030, 460 (243, 709.

After every ſix Figures beginning from the right Hand, make a lunula (, or a daſh |, to divide the Number into ſeveral Members, every one of which will conſiſt of ſix Notes, except the laſt, which may have leſs than ſix, even but one. Then expreſs every Member, beginning at the left hand, as if they were alone, as has been already taught, but adjoin to every Member the value belonging to it, which the Table underneath ſhews.

Member.	Value.	Member.	Value:
Firſt.	as it lyes.	Fourth.	Trilion.
Second.	Million.	Fifth.	Quatrilion.
Third.	Bilion.	*&c.*	*&c.*

Therefore the Number D muſt be thus expreſſed.

Memb. 4. Ninty Six Thouſand, Six Hundred and Thirty Eight Trilions.

Memb. 3. Nine Hundred Eight Thouſand and Three Bilions.

Memb. 2. Thirty Thouſand Four Hundred and Sixty Millions.

Memb. 1. Two Hundred Forty Three Thouſand Seven Hundred and Nine.

When a Number hath many Cyphers uninterrupted towards the right Hand, the expreſſing thereof will be ſhorter; as if the Number E were given (which is greater than the Number of Sands contained in the Globe of the Earth.)

E 1 (000000 (000000 (000000 (000000 (000000 (000000.

Becauſe its laſt (that is the ſeventh) Member conſiſting of a Unite is only Significant, you will expreſs the whole Number by ſaying One Sextilion.

But if omitting the Words Bilion, Trilion, Quatrilion, *&c.* you had rather expreſs the Number by Millions only, having made the ſame Partitions as before, expreſs every Member as if it were alone, adding the Word Million ſo often as there are other Members preceeding that which you are about to expreſs, ſo the Number D will be expreſſed by Millions thus;

Memb. 4. 96. 638 Millions of Millions of Millions.

Memb. 3. 908. 003 Millions of Millions.

Memb. 2. 030. 460 Millions.

Memb. 1. 243. 709.

And the Number E will be expreſſed thus; One Million of Millions of Millions of Millions of Millions. The firſt way is ſhorter and moſt expedit, the other moſt uſed.

The eaſieſt way of all, but far longer than the other, is to expreſs Numbers by Thouſands only, thus; After every three Figures beginning at the right hand

hand ſet a Comma; then expreſs every Member, beginning at the left hand, as if they were alone, and add the Word Thouſand or Thouſands, ſo often as there are other Members præceeding that to be expreſſed. 1. Let the Number F be given to be expreſſed by Thouſands; the laſt Member is 96 Thouſand Thouſand Thouſand Thouſands, the laſt but one, 638 Thouſand Thouſand Thouſands.

F. 96, 638, 000, 000, 000.

The Reaſon of all which is manifeſt from the firſt Chapter.

The Writing of any Number is performed without any trouble, by help of the Table in the foregoing Chapter; as if this Number were given to be wrote in Figures, Forty Five Millions Seven Thouſand. In the Table you will find Tens of Millions to be in the eighth place, Millions in the Seventh, and Thouſands in the Fourth; therefore write 4 in the eighth place, 5 in the ſeventh, 7 in the Fourth, and fill up the vacant Places with Cyphers.

CHAP. III.

Poriſms, on which the Reaſons of Arithmetical Operations depend.

PORISM I.

IF one Figure be placed before a number (A) conſiſting of many Figures, its value is encreaſed ten fold, if two, a hundred fold, if three a thouſand fold; and ſo continually in a tenfold Proportion.

Demonſtrat. For ſeeing that by placing one Figure before the number A, every Figure thereof is advanced a place higher, ſo that

A 243	243	0
	243	00
	243	000

3 ſtandeth in the Second, 4 in the third, and 2 in the Fourth place; and thence the value of each Figure increaſes Ten-fold, as is manifeſt from the firſt Inſtitution of Arithmetick, explained in the Firſt Chapter; Therefore the value of all the whole number A is increaſed Ten-fold: Likewiſe if Two Figures be ſet before the Number A, every Figure thereof aſcends two places higher; therefore the value of each Figure, and thence the value of all the Number is increaſed a Hundred Fold, or a Hundred Times, *&c.*

Corollary. Hence its manifeſt that if before Two Numbers a and b, are ſet equal Numbers of Cyphers to make them c and d; then c and d ſhall be Equi-multiplices of a and b: For if before each be placed one Cypher, then c and d ſhall be Ten-fold of a and b, or contain a and b ten times; if Two Cyphers a Hundred times, *&c.*

a 32	3200	c
b 7	700	d

PORISM II.

A 1000
B 999

Of two Numbers, that is the greateſt which hath the moſt Figures in it.

This is manifeſt from the firſt Inſtitution of Figures; ſo A is greater than B.

PORISM III.

C 2000
D 1999

Of Numbers, containing equal Numbers of Figures, that is the greateſt which hath the laſt Figure greateſt.

C 34299
D 34390

So C is greater than D, as is manifeſt from the firſt Inſtitution of Figures.

PO-

PORISM IV.

IF two unequal Numbers A,B, *have equal Numbers of Figures, the greater does not contain the leſſer Ten Times.*

Demonſtration. To the leſſer Number B add a Cypher, and it makes the Number E; by the 2 Poriſm E is greater than A; but E contains the leſſer Number B ten times, by Poriſm the Firſt; Therefore A does not contain the leſſer B ten Times, which was to be demonſtrated.

A	2999
B	2000
E	2000|0

PORISM V.

THere are two Numbers given, whereof A *is the greater, and* B *the leſſer: The greater exceeds the leſſer by one Figure; but the laſt Figure* (3) *of the leſſer Number* B, *is greater than the laſt Figure of the greater Number* A: *I ſay the leſſer is not contained ten times in the greater.*

Demonſtration. To the leſſer Number B add a Cypher, and it will be E; by Poriſm 3, E is greater than A; but E containeth the leſſer Number B preciſely ten times by Poriſm 1. Therefore the greater A does not contain the leſſer B ten times. W W D.

A	2999
B	300
E	300|0

PO-

PORISM VI.

TWO Numbers A *and* B *are given; if the first Figure* c *of one of these Numbers multiplieth any Figure of a higher value, in the other Number as* d, *but taken simply; and the Product* e *be set in the place of the Figure* d, *and hath so many Figures placed before it as are before* d, *so as to make* f; *I say that* f *is the true Product, made by the Figure* c, *multiplying* d *according to its local value.*

d
3, 042 A
c
517 B
e 21
f 21. 000
3. 000. g
7 c —— z

Demonstration. Let *g* be put for the Figure *d*, estimated according to the value of its place; then $g \times c = z$; now it is required to shew that $f = z$. because $c \times d = e$ and $c \times g = z$ Therefore by *Euclid.* 17. 7. $d \,..\, g :: e \,..\, z$; also $d \,..\, g :: e \,..\, f$ (for seeing *g* and *f* are the Numbers, *d* and *e* equally advanced or increased in their value; that is, having equal number of Figures placed before them; it is manifest from the Corollary to Porism 1. that g and f are Equi-multiplices of the Figures d and e) therefore $e \,..\, z :: e \,..\, f$, whence by Euc. 9. 5. $z = f$, which was to be Demonstrated.

PORISM VII.

LET there be two Numbers A *and* B *given; if any two Figures* k *and* d *of each Number, simply taken, be multiplied one into another, and the Product* c *be advanced to such a place, as is compounded of the places of the multiplying Figures; that is, having so many Figures placed before* c, *as are both before* k *and* d: *I say that* f *is the true Product made by* k *and* d, *estimated according to their local value.*

De-

Demonstration. Let *h.* and *g.* be put for the Figures, *k* and *d* estimated according to the value of their places, then *h* x *g* = *z*; now it is required to shew that *f* = *z*; before *c.* the Product of *k* x *d* put so many Figures as are before the Figure *d*, so as to make *m*, by the precedent Porism *m* shall be the Product of *k* multiplyed into the Figure *d* estimated according to its places value, that is *m* = *k* x *g*; Now because by Construction *m* = *c* with so many Figures placed before it as are in the number *A* before *d*, and by Hypothesis *f* is the same with *c*, but with so many Figures placed before it, as in the numbers *A* and *B* are together before *d* and *k*, it's manifest that the Figures which in *f* are put before *c*, exceed those which in *m* are put before *c*, by so many Figures as are in *B* before *k*, that is, by so many Figures as are in *h* before *k*, seeing *h* is the same with *k* estimated according to its local value: Therefore by Corollary to Porism 1. *h* and *f* are equi-multiplices of *k* and *m*, and thence *k*.. *h*:: *m*.. *f*; and because as above *k* x *g* = *m*, and by Hypothesis *h* x *g* = *z*; therefore by Eucl. 18. 7. *k*.. *h*:: *m*.. *z*. wherefore *m*.. *f*:: *m*.. *z*, therefore by Eucl. 9. 5. *f* = *z*; which was to be demonstrated.

	d			
	3. 042.	*A*		
	k			
	5. 17.	*B*		
c. 15.	*m.* 15. 000			
f. 15. 000. 00				
k. 5	5. 00. *b*			
	g. 3. 000			
	c		c	
m. 15. 000.			15. 000. 00 *f*	
			z	

From this and the foregoing Porism, Multiplication is demonstrated in Chap. 7.

PORISM. VIII.

A B (60 C
360,5897 (6 d
(6,6000 f

Let there be two Numbers AB, *and* C. *given; if* C *divideth the Member* A *ſimply taken; and the quotient* d *be advanced to the place of the Member* A, *that is, if before* d *there be placed ſo many Figures as are before the Member* A, *as* f; *I ſay that* f *is the true quotient; which is had by dividing the Member* A, *eſtimated according to the value of its place, by the Diviſor* C.

A g
360 360,0000
C
60 6. 0000 f
z

Demonſtration: Let *g*, be put for the Member *A* eſtimated according to the value of its place; then *C* dividing *g*. quotes *z*. it is required to ſhew that $f=z$. becauſe *C* dividing *A* quotes *d* and *C* dividing *g* quotes *z* therefore by *Ax.* 8. *Eucl.* 7. $A=C \times d$ and $g=C \times z$ therefore by *Eucl.* 18. 7. $A..g::d..z$, alſo $A..g::d..f$ (for ſeeing *g* and *f* are the ſame with the Numbers *A* and *d*, encreaſed to equal places, that is, having by Hypotheſis an equal number of Figures placed before them, it is manifeſt from Corrol. poriſm 1. that *g* and *f* are equimultiplices of *A* and *d*) hence $d..f::d..z$; therefore by *Eucl.* 9. 5. $f=z$ W. W. D.

From this Poriſm Diviſion is Demonſtrated, Chap. 9.

PO-

PORISM. IX.

THe Artifice of Arithmetical operations, that is of Addition, Subſtraction, Multiplication *and* Diviſion, *conſiſts herein; that while we work, neglecting the value of places; Figures are taken only according to their ſimple value. Nevertheleſs, ſo as the Sums in* Addition, *the remainders in* Subſtraction, *the products in* Multiplication, *and the Quotients in* Diviſion *are ſet in their right places*; whence it follows that every Figure hath its due value according to its place.

PORISM X.

ONe ſingle Figure multiplying or dividing any Number, encreaſes or leſſens the ſame but by one Figure or place

Let there be given any Number, as *A*, this multiplyed by 10 is encreaſed but by one Figure, Degree, or place higher than it was at firſt, as is manifeſt from Poriſm 1. Therefore a number multiplyed by a ſingle Figure is not encreaſed more than one place higher, and the leſſer the Multiplyer is, the leſſer is the Product.

A. 365
10
3650

A 365 (36 5/10
10

Again, if 10 divides *A*, it ſhall not leſſen it more than one Degree or place, Therefore a ſingle Figure, *&c.* and the leſſer the Diviſor is, the greater the Quotient ſhall be.

CHAP.

CHAP. IV.

Addition.

Addition is the Collection of many Numbers into one Sum.

OPERATION.

9706	3. A
800	2. B
504	1. C
11010 6	D

Let the numbers A B C. be given to be added together, or Collected into one Sum.

Write the Numbers so under one another, that the first Figure may be under the first (that is, Units under Units) the second under the second (that is, Tens under Tens) the third under the third (that is, Hundreds under Hundreds) and so on.

Then having drawn a line under them, add the Figures of the first place together; and if the number composed of these consist but of one Figure, write it under the line in the first place; but if it consist of two Figures, the first of them only must be wrote under the line, and the other must be reserved to be placed under the next place; if of three which seldom happens, the second belongs to the second place, and the third to the third.

Next add together the Figures of the second place with that which was reserved, and write the number composed of them under the line in the second place, minding the caution already given: after the same manner the Addition of the remaining places is performed.

These precepts will be made more plain by an Example.

Ex-

Example.

97063	A
8002	B
5041	C
110106	D

The Figures of the first place, 1, 2, 3, make 6. which I write under the Line; the Figures of the Second place, 4, 0, 6, make 10, which number becauſe it conſiſts of two Figures, I write the first Figure 0, under the Line; the ſecond 1 I reſerve to be placed under the third place. In the third place I find not any ſignificant Figure, wherefore I write the reſerved Figure 1 under the Line in the third place. In the fourth place I find 5, 8, 7, which together make 20, which Number conſiſting of Two Figures, I write the firſt 0 under the Line in the fourth place; but reſerve the ſecond Figure 2, for the next place. In the fifth place I find 9, to which I add the reſerved Figure 2, and they make 11, which I write in the fifth and ſixth place as they lye; and thus the Operation being ended, the Number D is got, which is the Sum of the given Number A, B, C.

If there be many ranks of Numbers to be added together, it will be beſt to divide them into three or four Claſſes, by putting lines between them, and to get the Sums of every Claſs, which being afterwards brought to one, will give the Sum of all the Sums.

Demonſtration.

The Demonſtration is gathered from Poriſm 9. Chap. 3. for although in Arithmetical Operations, the local value of the Figures is neglected for a time, yet it was reſtored, while the Sums of every place were ſet in their places, as is manifeſt from th e Operation it ſelf.

Of

Of divers Species.

IF divers Species or Numbers of ſeveral Names, as Pounds, Shillings and Pence are to be added together, write like Species, or numbers of one Name, one under another, and the leaſt, to wit, Pence in the firſt place, and the other in order; then out of the ſum of the leſſer Species, which is 19 Pence, take the next Species, to wit, a Shilling ſo often as you can, and write the remainer below the Line under Pence, and reſerve the 1 Shilling taken out, to be added to the next Species, that is, to Shillings. The next Species being added together make 27 Shillings, to which adding 1 Shilling taken out of the firſt Species the Sum will be 28 Shillings, out of this may be taken 1 Pound, which I add to the next Species, that is, to the Pounds, and the remaining 8 Shillings I write under the Shillings. The Sum of the third Species is 25 pound, to which add 1 taken out of the foregoing, Species and the Sum will be 26 Pound, which write under the Pounds, and ſo the Sum will be 26 Pound 8 Shilling and 7 Pence.

l.	*s.*	*d.*
9	17	6
8	04	3
5	02	2
3	04	8
26	08	7

After the ſame manner, numbers of any other Species may be added together, as is done in the following Examples,

Example 1.

l.	*oun.*	*dr.*	*gr.*
23	07	16	13
47	10	15	07
16	04	11	08
15	05	11	11
24	11	18	12
98	4	13	3

Example 2.

tun.	*hun.*	*qu.*	*l.*	*oun.*	*dr.*
12	11	3	18	11	15
12	10	2	15	10	10
12	09	1	12	10	14
9	09	3	13	11	11
9	08	1	10	10	10
56	10	0	15	7	12

Ells

Examp. 3.

Yds.	*qu.*	*na.*
24	3	2
34	3	1
21	2	2
19	1	3
100	3	0

Examp. 4.

Ells	*qu.*	*na.*
15	3	2
16	4	3
09	2	1
23	3	2
65	4	0

Having thus ſhewn how to add together ſeveral Integral Numbers, and alſo Numbers of diverſe Species, or Denominations, it remains for the better helping the young Learner to give ſome account of the ſeveral Diviſions, the Money, Weight, and Meaſure of this Nation are divided into; and how many of the leſſer Species is contained in the next greater, the knowing whereof will be alſo very neceſſary, both in Multiplication and Diviſion, for the reducing of the greater Species into the leſſer, and the leſſer into the greater, and will be of general uſe throughout all this Treatiſe of *Practicable Arithmetick*.

Money. The moſt uſual Coins uſed in *England*, are Pounds, Shillings, Pence, and Farthings; whereof 4 Farthings make a Penny, 12 Pence a Shilling, and 20 Shillings a Pound Sterling, Charactered thus, *l. s. d.*

There are three Sorts of Weight commonly uſed in *England*, as,

Troy weight. 1. *Troy Weight*, uſed in the weighing of Bread, Gold, Silver, *&c.* the uſual Species of which are, Pounds, Ounces, Penny-weights and Grains, whereof 24 Grains make a Penny-weight, 20 Penny-weights make an Ounce, and 12 Ounces a Pound, Charactered thus, *l. oz. pw. gr.*

Averdupois 2. *Averdupois Weight*, which is ſub-divided into little and great.

Averdupois little weights. *Averdupois little Weight*, is uſed in the weighing of Sugar, Pepper, Cloves, *&c.* by the Retail, the uſual Species of which are Pounds, Ounces, and Drams, whereof 16 Drams make an Ounce, and 16 Ounces a Pound; Characters thus, *l. oz. dr.*

Averdupois great Weight *Averdupois great Weight*, is uſed in the weighing all Commodities that are ſold by the Hundred, as Currants, Woolt, *&c.* the uſual Species of which are Tuns, Hundreds, Quarters of a Hundred, Pounds and Ounces; whereof 16 Ounces make a Pound, 28 Pounds a Quarter of a Hundred, 4 Quarters, or 112 Pound an Hundred Weight, 20 Hundred Weight a Tun, Characters thus, *T. C. qr. l. oz.*

Apothecaries Weight. 3. *Apothecaries Weight*, Divided into Grains, Scruples, Drams and Ounces; whereof 20 Grains make a Scruple, 3 Scruples a Dram, 8 Drams an Ounce, and 12 Ounces a Pound, Characters thus, *l. oz. dr.* ℈ *gr.*

There are ſeveral Sorts of Meaſures uſed in *England*, ſome Liquid, ſome Dry, and ſome relating to the meaſure of Time and Motion, *&c.* which take as follows.

Liquid Meaſure. 1. *Liquid Meaſure*, uſed in the meaſuring of all Liquid Subſtances, the ſeveral Species whereof are, Tuns, Pipes, Hogſheads, Gallons, Pottles, Quarts, and Pints; of which 2 Pints make a Quart, 2 Quarts a Pottle, 2 Pottles a Gallon, 63 Gallons a Hogſhead, and 2 Hogſheads a Pipe, 2 Pipes or Butts a Tun; Characters thus, *T. P. hhg. gll. Pot. q. p.*

Dry Meaſure. 2. *Dry Meaſure*, uſed in the meaſuring of all Dry Subſtances, as Corn, Salt, Coles, Sand, *&c.* the ſeveral Species whereof are Weys, Chaldrons, Quarters, Buſhels, Pecks, Gallons, Pottles, Quarts, and Pints, whereof 2 Pints make a Quart, 2 Quarts a Pottle, 2 Pottles a Gallon, 2 Gallons a Peck, 4 Pecks a Buſhel Land Meaſure, 5 Pecks a Buſhel, Water meaſure, 8 Buſhels a Quarter, 4 Quarters a Chaldron, and 5

Quar-

Quarters a Wey, Charactered thus, *W. Ch. Qr. Buſh. Pec. Gall. Pott. Qr. Pt.*

3. *Long Meaſure*, uſed in the Meaſuring of Land, Board, Glaſs, *&c.* the ſeveral Species whereof are, Miles, Furlongs, Perches, Feet, Inches, and Barly-Corns; of which 3 Barley-Corns make an Inch, 12 Inches a Foot, 3 Feet a Yard, 5½ Yards, or 16½ Feet, a Pole or Perch, 4 Perches a Furlong, 8 Furlongs an Engliſh Mile. *Long Meaſure.*

4. *Cloth Meaſure*, uſed in the Meaſuring of Woollen, and Linnen Cloth, *&c.* the ſeveral Species whereof are, Yards or Ells, Quarters and Nails; whereof 4 Nails make a Quarter, 4 Quarters a Yard, and 5 Quarters an Ell Engliſh, 3 Quarters an Ell Flemiſh; Charactered thus, Yds. Ells. Qrs. N. *Cloth Meaſure.*

5. *Time*, conſiſteth of Years, Months, Weeks, Days, Hours and Minutes; whereof 60 Minutes make an Hour, 24 Hours a Day, 7 Days a Week, 4 Weeks or 28 Days, a Month, 13 Months a Lunar Year. *Time.*

CHAP. V.

Subtraction

TEacheth to take or ſubtract a leſſer Number from a greater, and to ſhew the remainder, which may be done two ways.

Practice of Method 1.

WRite the leſſer Number under the greater, ſo as the firſt Figure may Anſwer to the firſt, the ſecond to the ſecond, the third to the third, *&c.*

Then draw a line under the two Numbers, and take the first Figure of the lower Number from the first of the upper one, and write the remainder under the line in the first place; then take the second from the second, and write the remainder in the second place, and so proceed: but if in the upper Number there remain any Figures, which none in the lower Number answer to, write them also under the line.

If any one of the inferior Figures be greater than the upper, to the uppermost add 10 in your mind, and estimate the next significant Figure, to the left hand in the upper Number, less by a Unite than it is; and if there be one or more Cyphers between Significant Figures, they are all to be reckoned Nines.

Examples.

496 A
75 B
421 C

1. Let B be given to be Subtracted from A; take 5 out of 6, and there remains 1, which write under the Line; then take 7 out of 9, and there remains 2, which write under the Line in the second Place; now there is left in the upper Number the Figure 4, to which no Figure in the lower Number answereth, therefore write 4 in the third place; and the Remainder sought is C.

3004 D
2578 E
426 F

2. Let E be given to be Subtracted from D; because 8 cannot be taken out of 4, I add 10 in my Mind to 4; and then take 8 out of 14 and there remain 6, which I write under the Line; but because I added 10 in my Mind to 4, which was borrowed from the following Number, we must reckon the next Significant Figure 3, only as 2, and the intermediate Cyphers as so many Nines; therefore I take 7 out of 9, and write the remainder 2 under the Line; also I take 5 out of 9, and write the Remainder 4 under the Line; lastly, I take 2, not

not out of 3, but out of 2, and there remains nothing; therefore the Remainer ſought is F.

3. Let H be given to be Subtracted from G, becauſe 9 cannot be taken out of 3, adding 10 in mind to 3, I take 9 from 13, and write down the Remainder; wherefore 5 is made 4, and 00 are turned into 99; therefore 6 out of 9 and there remains 3, which I write down; then becauſe there are no more Figures of the lower Number H to be Subſtracted, I write the remaining Figures of the upper Number, which now are not 850, but 849 under the Line, and I ſhall be the remainder ſought.

85003	G
69	H
84934	I

4. Let it be required to Subtract L from M, becauſe 3 cannot be taken from 0, adding in Mind 10 to 0, I take 3 out of 10 and there remains 7; then becauſe in the Number K the next ſignificant Number is 1, that vaniſheth, and every Cypher is turned into 9, from which Subſtract, as above.

100000	K
51243	L
48757	M

Demonſtration of Method 1.

It is manifeſt from the Operation it ſelf, that in writing down the Remainders, each Figure keeps its local Value; what remains to Demonſtrate is, why, when the lower Figure cannot be taken out of the upper, the next ſignificant Figure towards the left hand is leſſened by a Unit, and the intermediate Cyphers changed every one into 9.

See the ſecond Example D E F, becauſe 8 cannot be taken out of 4. to 4 I add 10, borrowed from the following Number, which is 3 Thouſand; but it is plain, that if from three Thouſand I take

3004	D
2578	E
426	F

ten, there will remain two Thouſand nine Hundred and Ninety; now becauſe they are expreſſed in Figures thus, 2990, its manifeſt why in the upper Number, 3 is turned into 2, and the Cyphers 00 into 99.

But if any one ſhould ſay, if before the firſt Figure 4 of the Number D, be ſet two other Figures 96, and before the firſt 8 of the Number E, two, 91, then 4 will ſtand for 400, and 8 for 800, and then to Subtract 800 from 400, there muſt be added not 10 but a 1000, that ſo 800 may be taken from 1400; I anſwer, that it will come to the ſame thing, whether you take 8 out of 14, or 800 out of 1400, becauſe the Remainder 6 is written under the Line, in the ſame place which the Figures 4 and 8 belong to, to wit, in the third; whence it comes to paſs that the Remainder 6 being ſet in that place, ſtandeth for 600, that is, ſo much as would remain, if 800 were taken from 1400.

D	3004	96
E	2578	91
F	.426	05

Practice of Method 2.

This differs from the firſt only in this, that when the lower Figure cannot be taken from the upper, and therefore ten is added to it, there is no alteration made in the next upper Figures following; but the following inferiour Figure is increaſed by a Unit, or if no ſignificant Figure follow, then One or a Unite muſt be ſuppoſed to be ſet in the following place; this will be more plain by an Example.

Example.

Let L be given to be Subtracted out of K, 6 from 8 and there remains 2, 7 from 6 I cannor, therefore

fore I add ten, then 7 from 16, and there remains 9; now becauſe I added 10, the next Inferiour Figure 5 is to be increaſed a Unit, and ſo 5 becomes 6; again 6 from 0 I cannot, therefore I take 6 from 10, and there remains 4; but becauſe I cannot add 10, I ſuppoſe a Unit to be ſet in the next place, which is vacant, which Subtracted from 8 leaves 7, therefore the remainder ſought is M.

8068 K
576 L
——
7492 M

This Method for the moſt part is more expeditious than the former, yet that will be the moſt convenient, when in the greater Number many Cyphers follow one another.

Demonſtration of Method 2.

Let there be given two Quantities, AB the greater, and CF the leſſer; Increaſe the ſaid Quantities by the equal Quantities EA and IC. then if you take IF out of EB, the remainder LB ſhall be the ſame, as if CF was taken out of AB; this if applyed to Numbers, the reaſon of the Operation will preſently appear; for to add 10 to the upper Figure 6, ſo as to make it 16, is nothing elſe than to put a Unit after it in the next place (which here is the third) wherefore ſeeing after the Inferiour Figure 7, is alſo put a Unit, in the next place (which is alſo here the third) it is manifeſt that each Number is equally increaſed, and ſo the Remainder got after this manner, is the ſame which remains if the Number L, be taken from the Number K.

E—A—L—B
I—C—F

Diverſe Species.

If diverſe Species or Numbers of ſeveral Names, as Pounds, Shillings, and Pence, be given to be Sub-

tracted one from another, write the leſſer Sum under the greater, ſo as like may anſwer to like, and if like cannot be taken from like, borrow One, or a Unit from the next greater Species.

Example.

l.	*s.*	*d.*
25 :	12 :	10
10 :	14 :	11
14 :	17 :	11

Becauſe 11 *d.* cannot be taken out of 10 *d.* from the 12 *s.* I borrow one that is 12 *d.* and add it to 10 *d.* which makes 22 *d.* then 11 out of 22, and there remains 11; and again, becauſe I cannot take 14 out of 11, (having taken away one before) to 11 I add one Pound or 20 Shillings, borrowed from the 25 *l.* which makes 31; now 14 from 31 and there remains 17. Laſtly I Subſtract 10 *l.* from 24 *l.* (for 1 was borrowed before) and there remains 14; ſo the remainder ſought is 14 *l.* 17 *s.* 11 *d.*

Or thus.

Becauſe I cannot take 11 from 10 *d*, I borrow one Shilling or 12 *d.* which added to 10 makes 22, then 11 out of 22 and there remains 11, which done, I carry 1 that I borrowed to the next Species, and add it to 14, then 15 from 12 I cannot; therefore borrow 1 pound or 20 Shillings which added to 12 makes 32, now 15 from 32 and there remains 17. Laſtly I carry 1 that I borrow'd to pounds which add to 10 makes 11, then 11 from 25 and there remains 14; ſo will be had the remainder 14. 17. 11. the ſame with the former.

After the ſame manner any other Species may be Subtracted, as in the following Examples.

l. 07.

l.	*oz.*	*pw.*	*gr.*
423	10	12	11
324	11	11	14
98	11	0	21

C.	*qr.*	*l.*	*oz.*	*dr.*
431	3	17	11	13
249	2	18	15	14
82	0	26	11	15

Ells	*qr.*	*n.*
341	1	2
121	3	2
219	3	0

yds.	*qr.*	*n.*
213	2	2
198	3	3
14	2	3

Questions to Exercise Addition *and* Subtraction.

Question 1. WHat Number is that which being Subtracted from 4898 the remainder is 329. *Answer* 4569.

From 4898 Subtract 329, and there will remain 4569, which is the Number sought, for that being taken from 4898 there remains 329 as was required.

4898
329
4569

Question 2. What Number is that to which if three times 486 be added, the Sum will be 10000. *Answer* 8542.

4898
4569
329

Question 3. What Number is that, from which if 3489 be Subtracted, the remainder will be 4689. *Answer* 8178.

Question 4. Since Queen *Elizabeth* began her Reign, is 138 years in what year of our Lord was it. *Answer* 1550.

Question 5. A Man says that this present year 1688, he is 89 years of Age, what year was he born in. *Answer* 1599.

Quest.

Queſtion 6. Two perſons A and B are of ſeveral Ages, the Age of the Elder is 70 years, and the difference of their Age is 20, what is the Age of B?

Anſwer. 50 years.

Queſtion 7. A lent B, 345 *l.* 14 *s.* 6 *d.* and after ſome time B. repays A. 40 *l.* 13 *s.* 8 *d.* at one time, and 100 *l.* at another, and gives him a Bill to receive 89 *l.* 10 *s.* how much has B. paid A. in all, and what remains yet due to the ſaid A.

Anſwer. Paid in all 230 : 3 : 1
Unpaid. 115 : 10 : 10

Queſtion 8. A Linnen Draper ſold 6 pieces of Muſlains, each piece contained 24 Ells, 3 quarters and 2 Nails, and was to have 5 *l.* 18 *s.* 8 *d. per* piece, how many Ells *&c.* were there in all, and what were they all ſold for. *Anſwer* 149 Ells $\frac{1}{2}$ and ſold for 35 *l.* 12 *s.*

Queſtion 9. A man pays away a certain Sum of money and forgets to enter it into his Caſh Book, till a day or two after; at which time he forgets what the Sum was; but recollecting himſelf, he remembers that if to the Sum were added 4 Guineas, it would make a 100 *l.* ſo deſires to know from a young Scholar what the Sum was he paid away, it is required to know what Anſwer he will give. *Anſwer* 95 *l.* 14 *s.*

Queſt. 10. A Grocer buys 4 Hogſheads of Sugar, each Hogſhead weighed Groſs, 19 *C.* 3 *qr.* 18 *l.* and the Tare of each was 24 *l.* what was the neat weight of all the Hogs-heads. *Anſwer.*

To reſolve this Queſtion, you muſt know that Groſs weight is the weight of the Sugar and Cask taken together; Tare is the weight allowed for the Casks only, and the neat weight that of the Sugar lone; the like muſt be underſtood of all Garbelled Commodities.

Hence, the Queſtion muſt be reſolved thus, firſt write the Groſs weight of the 4 Hogſheads under one another, and get their Sum; then alſo get the Tare of all the Hogſheads, and Subtract the Tare from the Groſs, and the remainder ſhall be the neat weight required.

CHAP.

A D								C
1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

A	D
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18

B E

X

1	5	9	7	
2	10	18	14	1194
3	15	27	21	1791
4	20	36	28	2388
5	25	45	35	2985
6	30	54	42	3582
7	35	63	49	4179
8	40	72	56	4776
9	45	81	63	5373

Z

The Table set with Rods out

B	E
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18

Sqr	Cube
0 1	0 1
0 4	0 8
0 9	0 27
1 6	0 64
2 5	1 25
3 6	2 16
4 9	3 43
6 4	5 12
8 1	7 29

CHAP. VI.

Of the Pythagorian Table, ſerving for Multiplication and Diviſion.

THE Pythagorian Table (which is ſo called from *Pythagoras*, who is ſaid to be its Author) hath on the top, or in the uppermoſt Rank A C, the 9 Original Arithmetical Figures, 1, 2, 3, 4, 5, 6, 7, 8, 9 and under every one of them is ſet its Double, Triple, Quadruple, *&c.* to its Noncuple; the uſe whereof is very great, and is ſhewn in Chap. 7. and 9.

Now if the Columns AB and DE *&c.* of this Table be ſeparated one from another, ſo as that they may be mixed amongſt themſelves, or ſet together in any order; thence will proceed an admirable compendium, for Multiplication and Diviſion; the Lord *John Neper* Baron of *Merchiſton*, was the firſt who Invented and Publiſhed this Section of the Pythagorian Table; to whom both for this Invention, and that of the Logarithms, the Mathematical World are infinitely indebted.

The conſtruction of this Diſſected and moveable Table, is after this manner; let there be prepared of Braſs, thick Paper, or any other fit matter, ſeveral oblong and thin Rods as AB, which divide into nine equal Squares, and cut each Square except the uppermoſt, into two Triangles by drawing Diameters; ſo as the lower Triangle B N P may be towards the right hand; on one ſide of the firſt Rod write the firſt Column of the Pythagorian Table, to wit, 1, 2, 3, 4, 5, 6, 7, 8, 9, and on the other ſide, the ſecond Column 2, 4, 6, 8, *&c.* On one

one ſide of the Second Rod write the third Column of *Pythagoras*'s Table, 3, 6, 9, 12. *&c.* and on the other ſide the fourth Column, 4, 8, 12, 16, *&c.* and ſo on the other Rods, write the remaining Columns of *Pythagoras*'s Table; always obſerving that when the Number conſiſts of but one Figure, it muſt be wrote in the lower Triangle; if of two as 18, then the firſt Figure 8 muſt be wrote in the lower, and the ſecond 1 in the upper Triangle, By each of theſe Rods, more may be made of the ſame kind and form; but 6 of a ſort are enough to perform the greateſt Multiplications and Diviſions; beſides theſe there is one Rod more as X Z to be prepared; which is ſeen by the Figures. This laſt is always ſet to the left hand in every Multiplication and Diviſion, ſhewing what Rank each Number is in; which from its office, is called the exponent.

But for the orderly laying and keeping the Rods in their due poſition; ſome have contrived a little Table or Frame, a little broader than the length of the Rods, and about once and a half the length of the breadth; upon the ſuperficies whereof cloſe to one of the Edges is glewed, or otherwiſe faſtened the ſaid exponent Rod; under the end of which is alſo faſtened a ledge the length of the Frame or Table; which when thus made and finiſhed, is called a Tabellet, and is noted by X, Z.

The chief and principal Artifice of theſe Rods, and whence all the other Uſes ſpring, is, that they ſhew in almoſt a moment, any Number Multiplied by the 9 Original Figures, as if the Number 597 were given; let thoſe Rods that will make up this Number in their tops, be ſet together on the Tabellet aforeſaid. So the exponent Rod X Z will ſtand to the left hand, and the Number 597 will ſtand in the firſt or uppermoſt Rank, and under it eight other Ranks of Numbers, which are ſuch Multiplices of the firſt 597 (that is, contain 597 ſo many times) as the Number on the exponent denotes, as is Demonſtrated Chap. 8.

How

How to Read and Write out any Number, contained in any Rank on the Rods.

TO do this, obſerve three things. 1. That the firſt lower Triangle which is towards the right hand makes the firſt place, every intermediate Rhombus, every intermediate place; and the laſt upper Triangle, the laſt place; from which it will preſently appear how many Figures a Number conſiſts of, which is contained in any Rank on the Rods. 2. that the Figures which ſtand in one and the ſame Rhombus (as in N M) muſt be added together: But if when added together, they make a Number conſiſting of two Figures, then the ſecond Figure muſt be added to the following Rhombus or Triangle.

Example.

Let the *9th.* Rank be given to write out: In the firſt inferior Triangle I find three, which I write in the firſt place; in the following Rhombus, the Figures are 6 and 1 which added together make 7; therefore I write 7 in the ſecond place, in the following Rhombus are the Figures 8 and 5; which added together make 13, I write the firſt Figure 3 in the third place; the other 1 I carry to the following laſt Triangle, and add it to the Number found there, which is 4, and they make 5, which I write in the fourth place; therefore the ninth Rank hath in it the Number 5373.

The Reaſon of writing out the Numbers from the Rods thus; ſhall be ſhewn in the Demonſtration of Chap. 8.

CHAP.

CHAP. VII.

Multiplication.

ONE Number is said to Multiply another, when the Number Multiplyed is so often added to it self, as there are Unites in the Number Multiplying; and another Number is produced; Or thus, one Number is said to Multiply another, when the Number that is produced is to the Number Multiplyed, as the Multiplyer is to a Unit.

To perform the Multiplication of Numbers, consisting of many Figures expeditiously; you must readily know the Multiplication of single Figures, which by the help of *Pythagoras*'s Table, is done in a moment; as if it were required to Multiply 7 by 8, in the side AB seek 7; in the side AC 8. in the angle of their meeting is found 57, which is the product of 7 by 8.

Practice.

LEt there be given two Numbers A, B, to be Multiplyed one by another; if one of these Numbers consists of more Figures than the other; write that which hath fewest undermost; then Multiply every Figure of the upper Number A, by the first Figure of the lower Number B, and write the Products under the line, from the right hand towards the left; observing, that if the Product consists of two Figures, to write down the first only; the other keep in mind, and add to the next Product.

In

In like manner the other Figures of the lower Number B, must multiply every Figure of the upper Number A; but the Products must be written so, as the first Figure D of the second Product may stand in that place, which the Multiplying Figure is in, that is, in the second; the first E of the third Product, in that place which the Multiplying Figure 5 is in, that is in the third place, *&c.*

Then Collect the Products, C, D, E, into one Sum F, which shall be the Product sought, by the Multiplication of the Numbers A, B.

Example.

Let the Numbers A and B be given to be multiplied: First multiply the Number A by 7, the first Figure of the lesser Number B. then 7 times 2 makes 14; the first Figure whereof 4 I write under the line, the other to wit, 1, I keep in mind, 7 by 4 makes 28, to which I add the 1 kept in mind, and it makes 29, I write 9 in the second place under the line and reserve 2; 7 times 0 makes 0, therefore I only write the reserved Figure 2, in the third place; 7 times 3 make 21, which I write in the fourth and fifth place, as they follow.

3042	A
517	B
21294	C
3042.	D
15210..	E
1572714	F

After the same manner the Number A is Multiplied by 1, the second Figure of the Number B, producing the Product D, the first Figure 2 of which, is wrote in the second place, that is, under its Multiplying Figure which is 1; and so also the Number A is Multiplyed by 5, the third Figure of the Number B, and produces E, the first Figure 0 of which is wrote in the third place, that is under its Multiplying Figure 5, but the other are placed in order, in their following places.

Demonstration.

It is manifest from the 6 and 7 *Porisms* of *Chap.* 3. that all the Figures in the first Product C, have their value due to their respective places, also that the first Figure of the Product D and the first Figure of the Product E have the like, is manifest from the 6 *Porisin*; and that the remaining Figures of the Products D and E, have their due value according to their respective places, is manifest from *Porism* the 7*th.* all which will be very plain, if the Operation already prescribed be compared with the forementioned *Porisms.*

Compendiums of Multiplication.

12000 G
400 H
4800000 K

23 . . . . G
5000 . . H
115000 K

1. When the first Figures of the given Numbers G H, are one or more Cyphers following one another, the Multiplication is performed only between the significant Figures, and the Cyphers of each Number, being taken together, are set before that Product.

2340 M
1000 N
2340000 P

2. When one of the given Numbers as N, consists of Unity and Cyphers, you have the Product P, if to the other Number M be added all the Cyphers of the first N.

8013 Q
5006 R
48078 S
40065 T
40113078

3. If in the Multiplier R, one or more Cyphers following one another, are included between significant Figures, then neglecting the Cyphers or Cypher; Multiply the Number Q by the following significant Figure 5, and write the first Figure

gure of the Product under its Multiplying Figure, which is here 5, and is in the fourth place.

The Demonstration of these *Compendiums*, is also manifest from 6 and 7 *Porisms* of *Chap.* 3.

Corollary.

TO Multiplication belongeth that part of Arithmetick, wherein any Number of a higher Species being given, it is required to find how many of a lesser Species are contained therein, which is called Reduction Descending, because in Operation we go from a higher to a lower Species.

Practice.

TO Reduce Numbers of a higher Species to a lesser, Multiply the higher Number given by so many of the lesser Species, as are contained in one of the greater, and the Product shall be the Number required; as if it were required to know how many Shillings are in 30 *l.* you must Multiply the said Sum by so many Shillings as are in one pound, to wit, by 20 and the Product is 600 Shillings.

If it were required to know how many Pence there is in 489 *pound.*

Multiply 489 by 20 to reduce it first to Shillings, then Multiply the Shillings by 12, because 12 *d.* makes one Shilling, and the Product will be 117360 the Pence contained in the Number given.

If it were required to know how many pence there is in 48 *l.* 19 *s.* 10 *d.*

```
489
 20
----
9780
 12
----
1956
178
----
117360
```

48 : 19 : 10
20
979
12
1958
080
11758

Multiply the Pounds as before by 20, and to the Product add the odd 19 *s.* which makes 979 Shillings, Multiply this by 12, and to the Product add the odd 10 *d.* which makes 11758, which is the pence contained in the Number given.

After the same manner, any other higher Species may be reduced to Species of a lower kind.

The Demonstration of which is gathered from what is already delivered, and from the Definition of Multiplication.

SCHOLIUM.

TO Multiply from the left Hand to the right, is of great use in Division; *and is performed in this manner.* 1. *No Figure is reserved or kept in mind, which is very convenient.* 2. *When the last Figures are Multiplied together, if by Multiplying the rest, any Product happen to be two Figures, the Figure on the left hand must be wrote under that Figure which in the foregoing Product is the first, or on the right hand.*

Example.

183
6
688
41
1098

If 183 is given to be Multiplied by 6, then 6 times 1 is 6; therefore I write 6 under the line, 6 times 8 is 48, because this Product consists of two Figures, I write the left hand Figure 4 under 6 and 8 before 6; 6. times 3 is 18. I write 1 under 8, and 8 before 8, which being added together, the Product sought is 1098. The Demonstration is gathered from what hath been already said.

CHAP.

CHAP. VIII.

Multiplication by the Rods.

MULTIPLICATION is performed with great expedition, by the Rods of *Pythagoras*'s Table; the construction of which is shewn in *Chap.* 6. Now the excellency and great use shall be declared.

Practice.

Let the Numbers A and B be given to be Multiplied together. 1. Seek those Rods which have the Figures of the Number A at the top, which place *a* so together that the Number A may stand in the upper Rank, and before them set the exponent Rod XZ. 2. Then on the exponent Rod XZ seek 8 the first Figure of B the letter Number given, and write out the Rank answering thereto; to wit, the eighth, which set under the Line, and will be C; after the same manner, seek 4 the second Figure of the Number B, on the exponent Rod, and write out the Rank answering thereto, to wit, the Fourth, which will be D, whose first Figure 8 must be wrote under the second Figure of the first Product C, if the Number B had more Figures, they likewise being found on the exponent Rod would shew the other Ranks of Figures to be wrote out. 3. Lastly the Products, C, D being added together, shall give the Product of A by B sought.

597	A
48	B
4776	C
2388	D
28656	E

a See the Table Chap. 6.

Demonstration.

The Demonstration is easie both from the construction of the Rods delivered in *Chap.* 6. and from what has been Demonstrated of common Multiplication, for the Number 56 found in the Eighth Square of the first Rod, is 8 times 7 and got by Multiplying 7 by 8; but because 56 consists of two Figures, I write 6 the first Figure in the lower Triangle under the line, and reserve 5 the other Figure in the upper Triangle; also the Number 72 found in the Eighth Square of the second Rod, is 8 times 9, that is, 9 Multiplied by 8; but because 72 consists of two Figures, the second 7 belongs to the third place, but the first 2 to the second only, and the reserved Figure 5 belongs also to the same place; wherefore 5 and 2 must be added together; the Sum which is 7. write in the second place under the line, which is the reason why the Figure in the upper Triangle, must be always added to the Figure of the following Triangle; also why the Figures in the me Rhombus, belong to the same place, at least as to the first Figure of their Sum, for if the Collected Figures of any Rhombus make a Number consisting of two Figures; the second Figure is carried to the following Rhombus. Lastly the Number 40, found in the 8 Square of the third Rod, is made of 5 Multiplied by 8, whose first Figure 0 only belongs to the third place: to which because the reserved Figure 7 also belongs, standing in the same Rhombus it must be added; but because 0 added to 7 makes 7, write 7 in the third place, and the other, which is 4, and in the upper Triangle, write in the fourth place; therefore the Eighth Rank on the Rods, that is, the Number C, is that which is made by Multiplying A. 597 by 8 the first Figure of the Number B. By the same reason may be shewn that the fourth Rank, that is, the Number

Number D, is that which was made by Multiplying A 597 by 4, the second Figure of B, therefore their Sum E is the Number which is made by Multiplying A by B, W W. D.

Hence the chief Artifice of these Rods consists in two things: First, That the Rods may be so composed and put together, that any Number may stand in the upper Rank. Secondly, That Numbers consisting of two Figures, are so placed on the Rods; that the first Figure stands in the lower Triangle, and the other in the upper; whence it happens that because every Superior Triangle, with the Inferior triangle of the next following Rod, constitutes a Rhombus, the upper Figure of the first Rod (which is always that which is kept in mind) is found in the same Rhombus, with the first or lower Figure of the following Rod, with which it must be added to be wrote in the same place with it.

Moreover how expedite, easie, and secure this method is, any that are willing to experience it, will soon find by use.

CHAP. IX.

Division.

ONE Number is said to divide another, when the Number found or Quotient shews how often the Divisor is contained in the Dividend; or thus, one Number is said to divide another, when the Number found or Quotient is to a Unit, as the Dividend to the Divisor.

I shall give two different Methods of dividing, whereof the first is least used, but is the best; the other most used, but the difficultest.

Practice of Method 1.

Let the Number A C, be given to be divided by the Number Z.

```
Z ) A  B C ( X
597) 45 90 93 (769
     4179
     ----
   D 411, 9
      358, 2
      ------
     E 537, 3
       537, 3
       ------
           0
```

1. Out of the Dividend take so many of the last Figures as are contained in the Divisor; or if these make a Number less than the Divisor (as it happens in our Example, seeing 459 is less than Z, 597) put on more to it as 0; and so the first Member A B of the Division is made, which distinguish by a point, and call the Dividual.

2. See how often the Divisor Z is contained in the Member A B, or Dividual; if it do not appear plain, seek how often 5 the last Figure of the Divisor is contained in the last two Figures of the Member A B, (but if the Divisor and Dividual consist of equal Number of Figures, then must be sought how often the last Figure 5 is contained in the last of the Dividual) which you will find to be contained 7 times; therefore write 7 after the crooked line, or Lunula, but you must never write more than 9 at once, after the said Lunula, because, as I will Demonstrate below, the Divisor is never contained in the Dividual more than 9 times.

3. Multiply all the Divisor Z by (7) the Figure already wrote in the Quotient, from the left to the right Hand; the Product 4179, which must be either less than, or equal to the Dividual, write under the said Dividual and subtract it from the same, which done, the remainder is *D*,411, which must be less than the Divisor Z, and write it under the line placing a point before the first Figure thereof; and if all this succeeds the Fi-

Figure 7 wrote in the Quotient after the Lunula shall be the right Figure, and the Division of the first Member performed.

4. But if the Figure wrote in the Quotient, when it Multiplys the Divisor, exhibits a Product greater than the Dividual, reject that, and take a less in its stead; but if it gives a Product too small, that is such as when you have subtracted it from the Dividual, it gives a remainer greater than the Divisor; rejecting that also, a greater must be taken; which Tryal of the Figure to be set after the Lunula, is soonest performed by Multiplying from the left hand, towards the Right, and is shewn in the fore-going Chap.

Wherefore the Difficulty of Division chiefly consists herein, to write such a Figure in the Quotient, by which if the Divisor be Multiplied, the Product may be subtracted from the Dividual, and the remainder be either nothing, or less than the Divisor, for such a Figure will shew how often the Divisor is contained in the Dividual.

5. The Division of the other Members is performed after the same manner, to the right hand of the remainder D (411) of the first Member AB, set the Figure 9 which goeth before the first Member AB; and so the second Member or Dividual is 411, 9 the Operation of which must be like the first, *viz.* Seek how often the Divisor Z is contained therein, which you will find six times; then write 6 after the Lunula before 7, and Multiply the Divisor by 6, and set the Product 3582 under the dividual 411, 93 then Subtract one from the other, and place the remainder E 537, under the line, and set a point before it.

Lastly, To the remainder E, 537, write the Figure 3 which follows to the right Hand, and you have the third Member 537, 3. Again seek how often the Divisor is contained in the third Member or Dividual, which you will find nine times, therefore set 9 in the Quotient, and Multiply the Divisor by 9, the

the Product is 5373 which Subtract from the Member 5373 and there remains nothing.

This being done all the Division of the Number given AC by Z is ended.

Divis.
13) 51. 2 (39 $\frac{5}{13}$ N
39
12. 2
11. 7
5

6. If when the Division is ended, any thing remains (as in this Example there remains 5) write the remainder 5 over the Divisor 13, and draw a line between them, making the Fraction N; then the Integer placed after the Lunula, together with the Fraction N, shall be the Quotient.

5) 10, 4, 5 (209
10.
0. 4 | 5
4 5
0

7. If when the first Operation be done, it happens, that the Divisor 5 is not contained once in any of the other Members or Dividuals, (as 0. 4) then set a Cypher to the Quotient after the Lunula, and to that Member or Dividual set one Figure more of the Dividend, (as 5) a small dash being put between, that it may appear that the Member (0. 4 | 5) is made up of two, that so the Number of Dividuals may appear.

$\frac{3}{7}$ R

8. To divide a lesser Number by a greater as 3 by 7, the lesser must be wrote over the greater, drawing a line between them, to make the Fraction R, which is Demonstrated 2 *Book*, 2 *Chap.* 1 *Theor.* 2 *Corol.*

Demon-

Demonſtration.

There are ſo many Figures wrote after the Lunula, and ſo the Quotient X hath ſo many Figures, as there are Members or Dividuals in the Diviſion, as is manifeſt from the foregoing Operation; but there are ſo many Members as there are Figures preceding the firſt Member AB, in the Dividend AC, which is manifeſt in its ſelf, therefore the Quotient X conſiſts of ſo many Figures, as go before the firſt Member of the Dividend: Wherefore it is manifeſt from Poriſm 8. Chap 3. that the laſt Figure 7 wrote after the Lunula of the Number X, is the true Quotient of the firſt Member AB; that is, which tells how often the Diviſor Z is contained in the firſt Member AB; in like manner the ſame is ſhewn of the other Figures of the Number X wrote after the Lunula in reſpect of the reſt of the Members or Dividuals, therefore the Number X is the true Quotient procured by Dividing the Number AC by Z. W. W. D.

```
Z )  A  B     C ( X
597) 4590, 9, 3 ( 769
     4179
     ----
  D  411, 9
     3582
     ----
  E  537, 3
     537, 3
     ------
           0
```

If the Diviſor does not meaſure the Dividend, the true Quotient nevertheleſs is an Integer (39) with the Fraction (N) whoſe Numerator is what remains after the Diviſion is ended, and its Denominator the Diviſor it's ſelf which I thus Demonſtrate.

```
Diviſor
13) 512(39 5/13 N
    39
    --
    122
    117
    ---
      5
```

As

As a Unit is to the integral part of the Quotient; so is the Divisor to the Dividend less, the last remainder, that is, 1 .. 39 :: 13 .. 512—5; from the 1. *Theor Chap.* 2. *Book* 2. as a Unit is to a Fraction, so is the Denominator to the Numerator; that is, 1 .. N = $\frac{1}{13}$ :: 13 .. 5, by *Eucl.* 14. 5. as a Unit is to the Integer and Fraction, so is the Divisor to the Dividend; that is 1 .. $39\frac{5}{13}$ :: 13 .. 512: Wherefore according to the Definition of Division $39\frac{5}{13}$ is the true Quotient; which was to be Demonstrated.

Why more than 9 must never be wrote at once in the Quotient, is thus shewn; Either the Member or Dividual A, and Divisor B, consist of equal Number of Figures, or the Dividual exceeds the Divisor by one Figure, for it never exceeds it by more, as is manifest from the first Part: If therefore they have equal Number of Figures, it is apparent from *Porism* 4. *Chap.* 3. that B is contained less than ten times in A; but if the Member or Dividual A, exceed the Divisor B by one Figure, then the last Figure of the Divisor is greater than the last Figure of the Dividual, as is manifest from the first Part; and therefore also by *Porism* 5. *Chap.* 3. B is contained less than ten times in A; wherefore seeing the Divisor is contained in every Dividual less than ten times, the reason is manifest why we must never write more than 9 at once in the Quotient.

A. 2996
B. 1000

A. 2999
B. 300

Practice of Method 2.

Let the Number A be given to be Divided by the Number B.

1. Write the Divisor under the Dividend towards the left hand, so as the last Figures, and last but one, *&c.* of each, may answer one another; but if the Number 370, which is over the Division, be less than the same, as in *Schem.* 1, then the Divisor must

muſt be removed one place forwarder to the right hand, as is ſeen in the Example. But whatſoever is towards the left hand of the Diviſor, is reckoned as placed over the Diviſor, whence 3702 is over 821, and 37 is over 8, and 570 is over 2.

Schem. 1

A 37029 (4
B 821

Schem. 2

5
37029 (4
821

Schem. 3

4
52
37029 (4
821

Schem. 4

41
528
37029 (4
821

2. See how often the Diviſor is contained in the Figures over it; but becauſe it is very difficult for the moſt part to judg of this, ſee how often the laſt Figure 8 of the Diviſor, is contained in the Number 37, wrote over it, and you will find it to be 4 times, therefore write 4 after the Lunula.

3. Multiply all the Diviſor by the Figure 4 wrote in the Quotient, and Subtract the Product from the Figures 3702, which are wrote over the Diviſor; then write the remainder over the foregoing Figures, and Cancel the Diviſor and the Figures which were over it; but moſt chooſe to Subtract every Figure of the Diviſor multiplyed ſeparately by the Quotient 4, from the Numbers over them; that is, they multiply every Figure of the Diviſor diſtinct by the Quotient, and Subtract every particular Product of ſuch Multiplication from the Figures which are over it.

Example

Example.

a Sch. 1. *b Sch.* 2. *c Sch.* 3. *d Sch.* 4.

a 8 By 4 makes 32, which Subtracted from 37, there remains 5, Cancel *b* 37 and 8, write 5 over 7; again 2 by 4 makes 8, which Subtracted from 50 leaves 42. Cancel *c* 50, and 2 in the Diviſor, and write the remainder over 50; laſtly 1 by 4 makes 4, which Subtracted from 422, there remains 418; then Cancel *d* 422 and 1, and write the remainder 418 over 422; and thus one application of the Diviſor is performed.

4. But becauſe 8, the laſt Figure of the Diviſor is multiplied by the Quotient 4, and the Product Subtracted from 37; and that the laſt Figure but one (2) is alſo multiplyed by the ſame Quotient 4, and the Product Subtracted from the remainder 50 placed over it; and likewiſe, that the firſt Figure 1, multiply'd by the ſame Quotient (4) is alſo Subtracted from the Remainder 422 placed over it; therefore hence it is manifeſt, that when it is ſought how often the laſt Figure 8 of the Diviſor, is contained in the Figures 37 ſtanding over it, ſuch a Figure muſt be placed after the Lunula in the Quotient, by which all the Figures of the Diviſor being multiplied, the Product may be Subtracted from the Figure wrote over the Diviſor, and yet under this further Reſtriction that the laſt Subtraction being performed, the Remainder (418) may be either nothing, or leſs than the Diviſor, as is explained in *Sect.* 4. of *Method* 1. Wherefore, if when the firſt Subtraction is ended, the ſecond cannot be performed; or if when theſe are done, the third cannot, the Figure wrote after the Lunula muſt be leſſened by Unity, ſo often till every one of the Subtractions may be performed; therefore before you Work in earneſt, all the Operation in *Sect.* 3. muſt be wrought mentally, that the Figure wrote in the

Quo-

Quotient may be tryed, whether it be not too little, or too great.

We will explain what hath been ſaid above, by an Example.

Let the Number M be given to be divided by N; I ſeek how often N is contained in the Number 738 placed after it, which I find to be 3 times, therefore I write 3 after the Lunula; then 2 by 3 makes 6, which taken from 7, there remains 1; 3 by 4 makes 12, which taken from 13, there remains 1; 9 by 3 makes 27, which cannot be taken from 18 therefore the Figure 3 wrote in the Quotient is too big; wherefore rejecting that, in its place ſet 2, and repeat the tryal; 2 by 2 makes 4, which Subtracted from 7, there remains 3; 4 by 2 makes 8, which Subtracted from 33, there remains 25; 9 by 2 is 18, which Subtracted from 258, there remains 240, therefore the right Figure is 2.

Sch. 5.

1

M 738(32

N 249

Sch. 6

24

330

738(2

249

But although moſt mix alternately Subtraction with Multiplication, as is above explained; yet to me it ſeems the more adviſeable, to perform all the Multiplication together, and that alſo from the left hand to the right, always remembring the caution preſcribed in *Sect.* 4. *Method* 1.

5. The

5. The first Application of the Divisor being performed, as is shewn in *Sche.* 1, 2, 3, 4. remove the Divisor B one Figure towards the right hand, as you see done in *Sch.* 7, and the manner of working thorow all will be the same as was delivered before; and if there are more Applications to be made, they are to be performed after the same manner.

Sch. 7.

41
738
37829 (4
8211

6. If when the Divisor is removed, it cannot be contained once in the Figures placed over it, then write a Cypher in the Quotient, and remove the Divisor a place more forward, not Cancelling any Figure in the Dividend.

Sch. 8.

41
738
A 37829 (45
B 8211
82

7. And if any thing remains when the Division is ended, do therewith as is Prescribed in *Sect.* 6. *Method* 1.

In Sch. 8. *the whole Division of the Number A by B is exhibited, which before, or above, was exhibited in several* Sch. *only to make it the more clear and apparent.*

The Demonstration of this Method, is like the foregoing Demonstration.

But although this last way of Division is for the most part used every where, yet I esteem the first way to be much the better; for the second way Cancels all the Figures so, that it confounds the steps and members of the Division; so that if there should be any Error, it would be very difficult to Correct it, as is sufficiently apparent from *Sch.* 8. but the first sheweth distinctly every Member of the Division, the Product of every Multiplication, and

what remains in every Subtraction of the Products from each Member; whence the amendment is easie, if any error has happened.

The Use of the common Table of Pythagoras *in* Division.

LET there be given any Number consisting of two Places, as E, and any single Figure, as F, and let it be required to find how often F is contained in E.

Seek the Figure F in the side A B, and in the Numerical Line answering it, seek the Number E, or the next less than it, as (56) from this Number ascending to the side AC, you find the Quotient sought, as 7.

62 E
8 F
See Table in Chap. 6.

The reason hereof is manifest from the Construction of the Table in Chap. 6.

Compendiums in Division.

1. WHEN the last Figure of the Divisor D is a Unit, and all the rest Cyphers. From the Dividend C take so many Figures (829) as the Divisor hath Cyphers, and write those which are left (2889) after the Lunula, to which set the Figures cut off 829 wrote over the Divisor 1000, drawing a Line between them, and the whole Number 2889, with the Fraction $\frac{829}{1000}$ shall be the Quotient. But if the Figures which are cut off are also Cyphers, as in this Example, then those Figures only which remain to the left hand shall be the Quotient, to wit, 342.

C 2889 | 829
D 1000
G (2889 $\frac{829}{1000}$

Demonstration.

This is easily gathered from *Porism* 8. *Chap.* 3. and is also manifest from the second *Compend.* in *Multiplication, Chap.* 7. For 1000 multiplying 342, produceth 342000; therefore a 1000 dividing 342000 quotes 342. for from the *Def.* of Multiplication, *Chap.* 8. 1 .. 1000 :: 342 .. 342000. Wherefore by permutation, 342 .. 1 :: 342000 .. 1000. Therefore by the Def. of Division, 1000 dividing 342000, gives 342 for the Quotient.

342 | 000
1000 Divis.
(342
Quotient.

2. When the Divisor consists of one or more significant Figures, other than a Unit, and hath Cyphers to the right hand, from the Dividend A cut off so many Figures 46 to the right hand, as the Divisor hath Cyphers; and divide the remaining Figures 249, only by the significant Figures of the Divisor; if when the Division is ended any thing remains, as here there remains 1, set that with 46 cut off, over the Divisor, drawing a Line between, which place after the whole Number 124 in the Quotient; then the whole Number 124, with the Fraction M shall be the Quotient.

A 2, 4, 9 | 46
2
B 2 | 00
0. 4
0. 9 (124 $\frac{146}{200}$ M
8
1

The Demonstration hereof is deduced from *Porism* 8. *Chap.* 3. or from the first Compendium in Multiplication Chap. 7. by the like reasoning as above.

3. When

3. When the Dividend hath Cyphers to the right hand, and you have wrought till none of the ſignificant Figures remains, then write ſo many Cyphers in the Quotient, as there are Cyphers left in the Dividend, after the laſt Member or Dividual.

Let C be divided by D, 4 in the firſt Member 27 is contained 6 times; therefore I write 6 after the Lunula; 4 by 6 is 24, which I write under the Dividual 27, and being ſubtracted therefrom, there remains 3, which I write under the Line, and to it write the next Figure 2 of the Dividend; that I may have the ſecond Member 32, 4 in 32 is contained 8 times; I write 8 in the Quotient, 4 by 8 makes 32, which ſubtracted from 32, there remains nothing; and becauſe in the Dividend there remains nothing but Cyphers, I write ſo many in the Quotient, and the Diviſion is ended, which is Demonſtrated, as before.

C 27, 2, 000 (68000
24
D 4
3. 2
3. 2
0

CHAP. X.

Diviſion is performed very eaſily, by the Rods of Pythagoras *his Table.*

IN Diviſion the greateſt trouble is to find the Quotient by which the Diviſor is to be Multiplied; which Quotient, as alſo the Product of the Diviſor multiplied by the Quotient, the Rods do exhibit in an excellent Compendium: The reſt of the Diviſion is performed by the firſt Method in the precedent Chapter, which ſuits theſe Rods far better, than the ſecond in the ſaid Chapter.

Let the Number AC be given to be divided by the Number Z.

```
Z )  A B  C  ( X
597) 4590, 9, 3 ( 769
     4179
     -----
   D 411, 9
     358, 2
     -------
   E 537, 3
     537, 3
     -------
           0
```

1. Set the Rods together, ſo as they may have the Diviſor Z in their firſt and uppermoſt rank; beſides the Scheam placed here, view the Table Z X, in Chap. 6

2. Determine the firſt member A B, as in Sect. 1. Method 1. Chap. 9.

3. Seek what rank on the Rods, hath a number equal to the Dividual, or that is next leſs (but that rank is next leſs than the Dividual, that hath immediately following it a rank greater than the Dividual) and the number on the Exponent Rod denominating or pointing out the rank, is the Quotient, which write after the Lunula.

In our Example you will find the Rank next leſs than the Dividual to be the Seventh, therefore write 7 after the Lunula.

4. Write the Rank found out (which is the product of the Diviſor by the Quotient 7) under the Member or Dividual A B, and being ſubtracted therefrom, write the remainder (411) under the Line, to which ſet the Figure 9, that goeth before the firſt Member AB in the Dividend, and ſo you have the ſecond Member (411. 9.)

5. The Diviſion of the ſecond Member, and of all the following, muſt be performed after the ſame manner as the firſt.

6. If the Member or Dividual be leſs than the Diviſor, that is, than the firſt Rank on the Rods, you muſt write a Cypher in the Quotient; and to the Dividual muſt be placed one Figure more of the Dividend

vidend, and ſo you have a new Member or Dividual.

7. If the Ninth Rank be leſs than the Dividual, write 9 in the Quotient.

Finally, by a very ſmall Exerciſe, you will perceive in a moment which Rank on the Rods is next leſs or greater than the Dividual; alſo the 2 and 3*d. Poriſms* in *Chap.* 3. are to be called to Mind, as likewiſe what is before taught, in relation to the writing out, and reading the Ranks; but you muſt chiefly conſider the laſt quadrate of the Ranks, to wit, how great the Figure is which is ſet in the laſt Triangle to the left hand, and whether the Sum of the Figures in the following Rhombus, exceeds 9. and ſo whether any Figure is to be carried to the ſaid laſt Triangle.

Demonſtration.

The Demonſtration is gathered from what is Demonſtrated in Chap. 9. and from the Definition of Diviſion.

But none knows how excellent this way of Diviſion is, but they who have experienced it; for by this one may perform more Diviſions in an Hour or two, than in a whole Day by the common way; to which Compendium both of Time and Operation, you have alſo the ſecurity of working, without being ſubject to miſtake.

Corollary.

To Diviſion belongs that part of Arithmetick, wherein any Number of a lower Species being given, it is required to find how many of a greater Species are contained therein, which is called *Reduction aſcending*, becauſe in Operation we go from a lower to a higher Species.

Practice.

To reduce Numbers of a lesser Species to a greater, divide the given Number by so many of the lesser Species as are contained in one of the greater, and the Quotient is the required.

Example.

```
20)428|0(214
   4
   ---
   02
    2
   ---
    08
     8
    ---
     0
```

If it were required to know how many Pounds there are in 4280 Shillings.

Divide the Shillings by 20, because 20 Shillings make a Pound; and the Quotient shall be 214, the Pounds required.

It is required to know how many Pounds are in 433 Pence.

```
12)433 (36
   36  20)36(1
   ---    20
    73   ---
    72    16
   ---
     1
```

Divide the Pence first by 12, which quotes 36 Shillings, which Divide again by 20, and it will quote 1, the Pounds required, and 16 Shillings 1 Penny over.

The remainders of each Division are the odd Shillings and Pence over and above the Pounds, contained in the Number given, and are denominated from the Dividend, of which they are part; the like must be understood of other Species.

It

It is required to know how many Hundred weight are contained in 48978 Ounces.

Divide the Ounces by 16, the ounces in 1 pound, ſo you have 30601 *l* which Divide again by 28, the pounds in one Quarter of an Hundred; ſo you have 1092 qrs. of a C. which divide again by 4, the qrs. in an Hundred, and you have 273 Hundred: hence the Anſwer is 273 Hund. 0 qrs. 25 l. 2 oz.

```
oz. 28) l.
16) 48978 ( 30601 (1092
  48        28
  ---       ---
  097       260
   96       252
  ---       ---
    18      ..81
    16        56
    --      ---
     2        25
4)1092 (273
  8
  --
  29
  28
  --
  12
  12
  --
   0
```

Queſtions to Exerciſe Multiplication and Diviſion.

Q. 1. WHat Number is that which being Divided by 1423 quotes 48? *Anſwer.* *Reſolved by Multiplying* 1423 *by* 48.

Queſt. 2. What Number is that, by which if you divide 4834, the Quotient will be 342? *Anſwer.* *Reſolved by Dividing* 4834 *by* 342.

Q. 3. What Number is that, by which if you multiply 343, the Product will be 34878? *Anſwer.*

Q. 4. What Number is that, which multiplied by 232, the Product will be 4764? *Anſwer.*

Q. 5. The Circumference of the Earth is eſtimated 360 Degrees (60 miles making a Degree) how many Barley Corns are contained therein?

Q. 6. From *London* to *York*, is 2861200 Barley Corns; How many Miles is it? *Answer.*

Q. 7. A Gentleman gave 300 Purses of Money to his Daughters Portion; in every one of half the Purses, was 100 broad Pieces of Gold, valued at 25 *s.* In every one of the other half, was 200 Crowns, and 50 Guineas, English Coin; How many Pound Sterling was her Portion valued at? *Answer.*

Q. 8. A House was Built and Finished the first day of *March*, in the Year of our Lord 1620; How many Minutes has it stood to this present 14 day of *July*, 1688? *Answer.*

CHAP. XI.

The Proofs of Addition, Subtraction, Multiplication, and Division.

THE safest way is to prove these several Rules one by another, for all other ways are subject to Error.

To Prove Addition.

Addition is proved by Subtraction: Let X be the Sum made by the Addition, of the Numbers A and B; subtract either of the Numbers added, suppose B, from the Sum X, if the remainder C is the same with the other A, the Addition is right; if not there is an Error.

123	A
25	B
148	X
25	
123	C

But

But if you have a Sum Z, made by the Addition of three Numbers D, E, F, or more, *then the Proof is after this manner*; begin from the left hand, and say, 2, 8, and 4 make 14, which subtracted from 16, there remains 2, which with 0 makes 20; then 9, 3, 7, makes 19, which taken from 20, there remains 1, which with 8 makes 18; 7, 5, 6, make 18, which taken from 18 there remains nothing; therefore the Addition is right, seeing the Sum Z is equal to all its parts taken together.

297 D
835 E
476 F
1608 Z
210

To Prove Subtraction.

Subtraction is proved by Addition; Add the remainder X to the Number Subtracted H; then if their Sum K agreeth with the Number G, from which the Subtraction is made, the Subtraction is right; but wrong if not.

234 G
52 H
182 X
52
234 K

Also Subtraction may be proved by Subtraction, thus; Subtract the remainder X from G, and the new remainder will be L; if this agrees with the Number H first Subtracted, the first Subtraction was right; the reason of each Proof is plain.

234 G
52 H
182 X
52 L

To Prove Multiplication.

Multiplication is proved by Division; Let the Numbers multiplying one another be N, P, divide the Product Q, by the multiplier P, if the Quotient R agreeth with the Multiplicand N, the Multiplication is truly performed; if not, the contrary.

523 N
6 P
3138 Q
(523 R

The

The Reason is manifest from the first Definition of Multiplication and Division; if the Multiplication be performed by the Rods, there is scarce any need of a proof, that Method is so secure from error; yet if you are willing to prove it, there is no proof easier than to reiterate the Multiplication.

To prove Division.

B A
3) 24, 8(124 C
2
04
4
0.8
0.8
0

Divis.
E A
3) 24. 11(803 F
24
01,1
9
2

2409 G
2
2411 H

Division is proved by Multiplication; let the Divisor B be multiplyed by the Quotient C, if that produces the Dividend A, the Division is right; as is manifest from the Definition of Division and Multiplication: if after the Division any thing remains, multiply the Divisor E by the Quotient F, and to the Product G add the remainder 2. if the Sum H is equal to the Dividend A, the Division is true; But if the Division is performed by the Rods, there is scarce need of any proof; but if you have a mind to prove it, the Multiplication necessary for the proof is performed by the same Rods, resting in their places, which served for the Division.

An easie way of Division.

Besides the two Methods of Division already delivered, I will add one more, which to me seems the compleatest, the particular Multiplications being so dis-

disposed in operation, that from them the Division is proved without any new Multiplication or Writing out again the particular products aforesaid for Addition; and is thus.

Let the Number 4589 be given to be divided by 34.

```
      ~~1~~(3
     1~~1~~ ~~6~~(3
34) ~~4~~ ~~5~~ ~~8~~ ~~9~~ (134
    ---------
     3426
     103
       1
    ---------
     4589
    ---------
```

To the left Hand of the Dividend write the Divisor 34 making a Lunula between them, also make a Lunula to the right Hand of the Dividend for the Quotient, and draw a line; then seek as in Sect. 2 Method 1. how often 34 is contained in 45. which you will find to be 1. multiply the Divisor by 1. and write the Product under the line, and subtract it from the Figures in the Dividend over it, so there remains 11. which write over the said Dividend; so you have 118 for a new Dividual; seek how often the Divisor is contained in that, which will be 3 times, the Divisor Multiplyed by 3. makes 102 which I again write under the line and subtract it from the said 118 and write the remainder, if any, over it; then you have another new Dividual 169 in which the Divisor is contained 4 times, the Product of whose Multiplication being also wrote under the line, and subtracted from the Dividual, there remains 33. and so the Division is ended.

Now the Division is proved thus; add all the Figures wrote under the Dividend together, taking in the remainders, if any happen; and if the Product be equal to the Dividend, the Division is rightly performed; the contrary if not equal.

SCHO-

SCHOLIUM.

I Shall not here Write the ways of proving by casting away the Nines, because they are fallacious; yet shall shew the many excellent properties of the said Number.

1. Nine Measures every Number, whose Figures taken simply makes Nine, such Numbers are, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, 117, 126, and infinite others.

2. The Figures of every Ninth Number in a natural Series of Numbers, continued infinitely, from a Unit, being taken singly & added together make 9. those Numbers only excepted, whose Figures are all Nines, of which sort is 99, 999, &c.

3. If any Number consisting of two Figures, that 9 does not measure, be divided by 9. the remainder shall be the same with that which remains, if the Figures were taken singly and divided by 9; as if 64 were divided by 9 there remains 1; now if the Figures 6 and 4 are taken according to their simple value they make 10, which being divided by 9, leaves also 1; this is manifest from 1 and 2.

4. But if the Figures composing the given Number, being taken singly make less than 9, that which they make shall be the remainder which is left, if the given Number be divided by 9: Let 33 be given these divided by 9, there remains 6, that is, so much as the single Figures 3 and 3 make.

5. Wherefore from these it follows, that of every Number divided by 9 there remains the same, which remains when the Figures of that Number are taken singly and divided by 9.

I have not read any who have opened the Reason of these things, but I will shew the Fountain in a word; Nine is less than Ten by Unity; If you rightly weigh and apply this, it will not be difficult to deduce the Properties afore-mentioned.

Moreover, although this Property is both excellent and most true, yet the Proofs made from it are fallacious; for if from the Figures of all the Numbers to be added, simply

ſimply taken, 9 be caſt away as often as it can; that is, are divided by 9, and the remainder be reſerved; alſo if from the Figures of the Sum taken ſimply, 9 be caſt away as often as it can; and if this remainder is the ſame with the firſt remainder, the Addition is concluded to be rightly performed: But the Concluſion is falſe; for although for the moſt part it does not err, yet there may be Infinite caſes given, in which this Proof ſhall declare the Addition right when it is wrong; the reaſon is becauſe 9 (or any other Number) by dividing two unequal Numbers, may give one and the ſame remainder; which is manifeſt of its ſelf.

Example.

In the Numbers A, B, C, 3 and 5 are 8, and 8 are 16, caſting away 9, and there remains 7, which with 2 makes 9, which caſt away; 3 and 4 make 7, therefore the Numbers A and B, being divided by 9, there remains 7: Now in the Number C, 8 and 3 make 11, caſting away 9, there remains 2, which with 5 make 7; therefore in each part, both in the Numbers A B to be added, and in the pretended ſum C, there remains 7; yet C is not the true Sum of A B, for they make 592, In like manner both in Subtraction, Multiplication, Diviſion, and in the extraction of Roots the truth of this manner of proof is to be ſuſpected.

358	*A*
234	*B*
835	*C*

THE

THE SECOND BOOK OF PRACTICAL Arithmetick.

Of Fractions, *or* broken Numbers.

CHAP. I.

The Definitions of Fractions, *and how to Write and Expreſs them.*

A Broken Number, which is alſo called a Fraction and Piece, is a Part, or Parts of a Unit, repreſenting ſomething whole and diviſible; as if any whole were cut into five equal parts, and any ſhould take three of them, thoſe three fifth parts are called a broken Number.

Wherefore ſeeing a broken Number is the part, or parts of any whole, it muſt be wrote with two Num-

Numbers, whereof one may tell how many parts are taken out of the whole, and the other what parts; alſo thoſe Numbers are to be wrote one over another, drawing a Line between them, as you ſee done in theſe Examples.

A $\frac{1}{2}$ B $\frac{2}{3}$ C $\frac{4}{7}$ D $\frac{11}{13}$

The upper Number ſhews how many parts are taken out of the whole, or Integer; the lower ſhews into how many parts the whole is divided; and ſo what thoſe parts are, which are taken out of the whole.

Therefore the upper Number (becauſe it tells the parts taken out of the whole) is called the Numerator; but the inferior (becauſe it ſhews what manner of parts are taken) is called the Nominator, or Denominator.

The Fractions, A, B, C, D, are thus expreſſed; A one Second, or an Half; B two Thirds; C four Sevenths; D eleven Thirteenths.

But although a Unit for the moſt part is taken for the whole, or Integer that is Divided, yet it may be ſuppoſed to be any other Number; and ſo the Fraction E (if the whole or Integer be 64 Crowns) ſignifies five eighth parts of 64 Crowns. $\frac{5}{8}$ E

To underſtand the Nature of Fractions thorowly, this is carefully to be obſerved, that Fractions do not differ from whole Numbers in the main: The only difference is, that Fractions deſign things which are parts of things deſigned by whole Numbers; and therefore the Unities of a broken Number are relative, but the Unities of a whole Number are abſolute. For Example, Let 125 Pound be given, and the Fraction *a*, that is four Twentys of one Pound, the whole Number 125 contains a hundred and twenty five Units, whereof each is a Pound; likewiſe the Fraction *a*, that is, four twenty parts of a Pound, contains four Units, each whereof denote one twentieth part of a Pound, that is, a Shilling, and $\frac{4}{20}$ *a*.

and thence four twenties of a Pound ſignifie four Shillings, which is a whole Number, as well as 125 Pounds. Further, every whole Number deſigning the part of any thing is a Fraction, if it be compared with the greater whole Number, for 2 Shillings are the one tenth of a Pound, or *b*, and 3 Feet are the 3 fifths of a Pace or *c*, and 2000 Paces are the two fifths of one Italian Mile, or *d*. It concerns Learners to conſider theſe things very well; for the chief reaſon why the treating of broken Numbers uſeth to ſeem ſo difficult and obſcure to them, is that they run to learn the Operations and Rules of Fractions, before they underſtand their nature.

$\frac{1}{10}$ *b*
$\frac{3}{5}$ *c*
$\frac{2}{5}$ *d*

A General Obſervation.

When two or more Fractions are compared between themſelves, they are to be underſtood always parts of the ſame whole or Integer, unleſs the contrary be declared.

CHAP. II.

The Theorems of Fractions.

THEOREM 1.

EVery Fraction is to its Whole or Unity, as the Numerator is to the Denominator.

Demonſt. This is manifeſt from the Definition of Fractions explained in the preceding Chapter; for the Denominator denotes the whole cut into ſo many equal parts as there are Units in the ſaid Denominator: But the Numerator ſheweth how many parts are taken of the whole ſo cut. Therefore ſeeing a Fraction

ction is nothing elſe, cut the Number of Parts which the Numerator ſheweth to be taken of the whole; it is manifeſt, that the Fraction is to its whole, as the Numerator is to the Denominator; which being rightly underſtood thereon depend the following *Corollaries*.

Corollary 1.

WHEN *the Numerator is leſs than the Denominator, the Fraction is leſs than its Whole, or Integer.*

When the Numerator is greater than the Denominator, the Fraction is greater than its Integer. So the Fraction $\frac{30}{20}$ *a*, which is thirty twenties of a Pound (that is 30 Shillings) is greater than its Integer, that is, than a Pound, or 20 Shillings.

When the Numerator is equal to the Denominator, the Fraction is equal to its Integer. So *b*, that is 20 Twenties of a Pound (that is 20 Shillings) make a Pound; ſo *c*, *d*, are equal to a Unite, or to the whole, which a Unit doth repreſent.

$\frac{20}{20}$ *b*

$\frac{3}{3}$ *c*

$\frac{4}{4}$ *d*

Corollary 2.

FROM this Theorem is gathered the Truth of that which is delivered in the firſt way of Diviſion, Chap. 9. Sect. 8. *viz.* That when a leſſer Number (2) is to be divided by a greater (3) the Quotient is had; if the greater be wrote under the leſſer, ſo as to make the Fraction *f*, for by the Definition of Diviſion, Chap. 9. the Quotient is to Unity, as the Dividend, which here is 2, is to the Diviſor, which here is 3; but alſo the Fraction *f* is to Unity, as 2 to 3, therefore *f* is the Quotient.

$\frac{2}{3}$ *f*

THEOREM 2.

	E	F
a	2	3 c
b	4	6 d

IF as the Numerator a *of one Fraction is to its Denominator* b, *ſo the Numerator* c *of another Fraction, is to its Denominator* d; *thoſe Fractions are equal. Or thus, If* a .. b :: c .. d; *I ſay* E = F.

Demonſtration. By Theor. 1 E .. 1 :: a .. b; and by Hypotheſis, a .. b :: c .. d; wherefore E .. 1 :: c .. d; again by Theorem 1, F .. 1 :: c .. d; wherefore E .. 1 :: F .. 1: Therefore ſeeing E and F have one and the ſame reaſon to Unity, by *Eucl.* 5. 9. E = F; that is, the Fractions are equal to one another. W. W. Demonſtrated.

Wherefore the value of Fractions is not to be eſtimated by the greatneſs of the Numbers by which they are expreſt, but by their Proportion, which ſhall be further explained in Theor. 3, 4, 5.

THEOREM 3.

	A	B
e	8	2 g
f	16	6 h

THE Fraction A, *is greater than another Fraction* B, *whoſe Numerator* e, *hath greater reaſon to its Denominator* f, *than the Numerator* g *of the other, to its Denominator* h.

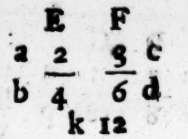
Demonſtration. For by Theor. 1. A .. 1 :: e .. f; again, B .. 1 :: g .. h; but by Hypotheſis, e .. f is greater than g .. h; hence alſo A .. 1 is greater than B .. 1; therefore by *Eucl.* 5. 10. A is greater than B; that is, the Fraction A is greater than the Fraction B. W. W. D.

THEO-

THEOREM 4.

THose Fractions are equal, whose Numerators being multiplied into their alterne Denominators, produce the same Number.

Let the Fractions E, F be given; if a x d = k, and also c x b = k. I say the Fractions E and F are equal.

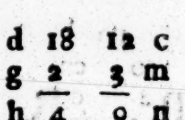


Demonstr. By *Hyp.* a x d = k, and c x b = k, that is, a x d = c x b, therefore by *Eucl.* 7. 19. a . . b : : c . . d; wherefore by *Theorem* 2. E = F. W. W. Demonstrated.

THEOREM 5.

THAT Fraction A is greater than another B, whose Numerator multiplying the others Denominator, produces a Number greater than that produced by its Numerator, being multiplied in the Denominator of the first.

Let g x n = d, and m x h = c, becaus d is greater than c, I say that A is also greater than B.



Demonstr. Because by Hyp. d is greater than c; therefore by Cor. 2. Prop. 19: Book 7. *Eucl.* g hath greater reason to h, than m hath to n: therefore by Theor. 3. of this Chap. the Fraction A is greater than the Fraction B. W. W. Demonstrated.

Corollary.

From the 4 and 5 Theorems is had an easie way to try whether Fractions are equal or unequal, and which is the greatest.

THEOREM 6.

FRactions (D, G) *having the same Denominator* (p) *have the same Proportion one to the other, as their Numerators* (m, n) *have; that is* D . . G : : m . . n.

Demonstr. By Theorem 1. D . . 1 : : m . . p; also G . . 1 : : n . . p; therefore by *Euclid* 5. 22. by equality of Reason, D . . G : : m . . n; which was to be Demonstrated.

D G
m 7/8 5/8 n
p p

THEOREM 7.

LET the Fractions P *and* R *be given, and let the Numerator* a *of the first multiplying the Denominator* o, *of the other, make* ao, *and the Numerator* c *of the other multiplying the Denominator of the first* n, *produce* cn; *then the Fraction* P *shall be to the Fraction* R, *as* ao *is to* cn; *that is* P . . R : : ao . . cn.

Demonstr. By Theor. 1. P . . 1 : : a . . n, and by *Hyp.* n x o = no, and a x o = ao; therefore *a* ao . . no : : a . . n; wherefore P . . 1 : : ao . . no; again because by *Hyp.* n x o = no, and n x c = nc, therefore *b* no . . cn : : o . . n, and by *Theor.* 1. R . . 1 : : c . . o; hence therefore by equality of Reason *c* P . . R . . 1 : : cn . . no R : : ao . . nc. W. W. Demonstrated.

a *Schol.* 10. *Prop.* 17. 7.
b *the same.*
c 22. 5.

no
ao cn
P a/n c/o R
} the whole.

THEOREM 8.

IF *two Fractions* (C, D) *have the same Numerator* (a) *the first* C, *and last* D *shall be in reciprocal Proportion with their Denominators* (o, n) *that is* C . . D : : o . . n.

De-

Demonſtr. Make a x o = ao, and a x n = an, than ao .. an :: o .. n, by the Preceeding *Theor.* C. .D :: ao .. an; therefore C .. D :: o .. n, W. W. Demonſtrated.

CHAP. III.

Reduction of Fractions.

PROBLEM I.

To Reduce a Fraction to its leaſt Terms.

A Fraction is ſaid to be reduced to its leaſt Terms, when another is found equal thereto, but expreſſed in leſs Numbers.

Let the Fraction A be given to be reduced to its leaſt Terms.

Conſtruction. By Prop. 35. *Eucl.* 7. find the two leaſt Numbers that have the ſame Ratio to one another, as the Numerator *n*, hath to the Denominator *p*, which let be *r* and *t*; then *n* .. *p* :: *r* .. *t*. I ſay the Fraction B, made of *r* and *t*, is that which is required.

	A	B
n 27		3 *r*
p 36		4 *t*

Demonſtr. By Conſtruction *n* .. *p* :: *r* .. *t*; therefore by Theor 2. of the foregoing Chapter, the Fraction A is equal to B; neither can there be given another Fraction in leſſer Numbers equal to the ſaid A; for, as is manifeſt from Theorem 2. to perform this, there are required two other Numbers in the ſame Proportion, as *n* to *p*, and leſs than *r*, *t*, already found, which is impoſſible to find.

But if the Terms *n*, *p*, of the given Fraction A, are the least in their Proportion, it cannot be expressed in lesser Terms; and whether the Terms *n*, *p*, are the least in their Proportion, is known by the very Construction, from *Euclid* 35. 7, and universally Numbers which are prime one to another are the least of all Numbers that have the same Proportion with them, by *Eucl.* 7. 23.

PROBLEM 2.

To Reduce Fractions to a common Denominator.

PART 1.

LET there be any two Fractions N and P given, to be reduced to a common Denominator.

Constr. Multiply the Denominators together, and the Product shall be the common Denominator, as $b \times d = bd$; then multiply the Numerators into their contrary Denominators, and the Products shall be new Numerators, to the new or common Denominators, as $a \times d = ad$, and $c \times b = cb$; I say then that the Fraction R = N, and T = P.

N		P	
a	3	5	*c*
b	4	6	*d*

ad	cb	18	20
bd	bd	24	24
R	T	R	T

Demon. Because by *Constr.* $a \times d = ad$, and $b \times d = bd$, hence by *Euclid* 7. 17. $ad \,..\, bd :: a \,..\, b$; therefore by *Theorem* 2. of the preceding Chap. N = R in like manner, seeing $c \times b = cb$, and $b \times d = bd$, it follows that $cb \,..\, bd \,..\, c \,..\, d$; and therefore P=T, wherefore the Fractions N, P, are reduced to a common Denominator, which was to be done.

PART

PART 2.

LET there be given three Fractions G, H, I, or more, to be reduced to a common Denominator.

Constr. Multiply all the Denominators b, d, f, one into the other, as b x d x f, and the Product bdf shall be the common Denominator. And multiply the Numerator of each given Fraction, into all the other Fractions Denominators, and the several Products shall be the Numerators to the common Denominator; so a x d x f = adf, c x b x f = cbf, and e x b x d = ebd, I say that the Fraction R = G, T = H, V = T:

G	H	I
$a\frac{3}{b\,4}$	$c\frac{5}{d\,6}$	$e\frac{1}{f\,8}$

R	T	V
$\frac{adf}{bdf}$	$\frac{cbf}{bdf}$	$\frac{ebd}{bdf}$

R	T	V
$\frac{144}{192}$	$\frac{160}{192}$	$\frac{24}{192}$

Demonst. By *Eucl.* 7. 17. adf.. bdf: :a..b therefore by *Theor.* 2. R = G again, because cbf.. dbf :: c..d; therefore T = H. Again because ebd.. fbd :: e..f; therefore V = I, and so the Fractions G, H, I, are reduced to the Fractions R, T, V, having one common Denominator, which was to be done.

The Letters shew all the Operation of each part at first sight, which excellent Property belongeth only to Specious Arithmetick.

PART 3.

LET there be any Number of Fractions, as A, B, C given to be reduced to a common Denominator, and in their least terms.

Con-

A	B	C
3	5	1
4	6	8

24

6	4	3
18	20	3
24	24	24
D	E	F

Conſtruction. By 36 and 38 of *Eucl.* 7. find the leaſt Number 24, which the given Denominators 4, 6, 8, will meaſure, and that ſhall be the common Denominator; divide this common Denominator by the Denominators 4, 6, 8, and by their Quotients 6, 4, 3, multiply the given Numerators 3, 5, 1, which ſhall produce 18, 20, 3, for the new Numerators.

Now I ſay, that the Fractions D, E, F, having one common Denominator, are equal to the Fractions A, B, C, given, and alſo are in their leaſt Terms.

Demonſtration 1. *That they are equal*, from the *Conſtruct.* and *Eucl.* 9. 19. it's manifeſt, 4 .. 24 :: 3 .. 18; that is, by Permutation 4 .. 3 :: 24 .. 18, and Inverſly 3 .. 4 :: 18 .. 24; wherefore by *Theor.* 2, A = D; by the ſame way of arguing, B = E, and C = F.

Demonſtr 2. *That they are in their leaſt Terms*; to get Fractions of the ſame Denominator, equal to the given Fractions A B C; of neceſſity the given Denominators 4, 6, 8, muſt evenly divide the common Denominator ſought, giving ſuch Quotients, which if multiplied into the Numerators given, may produce the new Numerators ſought, but by *Conſtruction* the common Denominator 24, already found, is the leaſt which the given Denominators 4, 6, 8, do evenly divide: Therefore there can be no other Fractions found, having a common Denominator, and in their leaſt Terms equal to the given Fractions A, B, C, other than D, E, F, which are already found.

PROB.

PROBLEM 3.

To Reduce a Fraction to a Denominator assigned.

LET the Fraction F be given to be reduced to another Fraction, whose Denominator is d.

Construction. By *Eucl.* 9. 19. to b, a, d find a fourth Proportional, which let be c, which with the Denominator d, makes the Fraction G equal to F, the Fraction given, as is manifest from *Theor.* 2.

F	G
a 2	8 c
b 3	12 d 12 d
	24 e

But if b do not measure or evenly divide e, as in this second Example, where b dividing e gives the Quotient n, m, write the Integral Part of the Quotient over the Denominator d, then the Fraction n, d, † m of p (that is six Eighths, with two Fifths of one Eighth) is equal to the Fraction a b.

Demonstr. Because b. . a : : d . . n, m; it is manifest that the Number n, m, declares how-many parts of d, that is how-many eighth Parts must be taken out of the whole, so as to have a Fraction, whose Denominator is d, equal to the Fraction a, b; seeing therefore every Unit of the whole Number n shews that so many eighth Parts are to be taken out of the whole; it is manifest that in the Nnmerator n, m, something is less than a Unit; to wit, that m sheweth somewhat else, less than one Eighth, must be taken out of the whole, *viz.* m p, that is two fifths of one Eighth, which is a Fraction of a Fraction; but more of these in Chap. 8.

		m p
a 4	8 d 6 n	2, 1
b 5	8 d	5, 8
	32 e (n	
	(6 ⅗ m	

The

7
20
8) 140 (17 4/8 or 1/2
8
60
56
4

The chief use of this Problem is, to know the value of any given Fraction in those parts which are most common and best known; as if it were required to know how many twenty parts of a Pound, the Fraction 7/8 of a Pound is; that is, how many Shillings are therein. It is found by the Rule thus; as 8 .. 7 :: 20 . . to that required; multiply 20 by 7, and divide by 8, and the Quotient gives 17 4/8, which are the Shillings equal to the Fraction given.

After the same manner may be found the value of any Fraction in the parts of the whole, best known and most common; that is by multiplying the Numerator of the Fraction given, by any parts assigned of the whole, and dividing the Product by the Denominator, for the Quotient gives the Number of Parts denominated, as was required.

Examples.

8
20
11) 160 (14
11
50
44
6

How many Shillings are contained in 8/11 of a Pound?

8 Multiplied by 20, produces 160, which divided by 11, quotes 14 Shillings, and 6/11 of One Shilling.

6
12
11) 72 (6
66
6

How many Pence are contained in 6/11 of a Shilling.

Multiply 6 by 12, the Pence in one Shilling; and the Product 72 divide by 11, and the Quotient is 6 Pence, and 6/11 of a Penny.

How

How many Pounds are contained in $\frac{30}{98}$ of a Hundred?

Multiply 30 by 112, the Pounds in a Hundred Weight; the Product 3360 divide by 98, and the Quotient gives 34 Pounds, and $\frac{28}{98}$ of a Pound, which by the ſame Method may be Reduced to Ounces.

112
30
98)3360(34
294
420
392
28

PROBLEM 4.

To Reduce a Fraction greater than its Whole, (or an Improper Fraction) to a whole Number.

LET the Improper Fraction be A; divide the Numerator b, by the Denominator c, and the Quotient d, ſhall be equal to the Fraction A.

A
b 12 (3 d
c 4

Demonſtr. By *Theorem* 1. Unity being put for a Whole, A . . 1 : : b . . c; and alſo by the Definition of Diviſion, d . . 1 : : b . . c; therefore by *Euclid* 5. 9. A = d which was to be Demonſtrated.

But if when the Numerator is divided by the Denominator, the Quotient is a whole Number, with a Fraction, as it happens in the Fraction B, then the given Fraction cannot be reduced wholly to a whole Number; and the whole Number and Fraction taken together, ſhall be equal to the given Fraction; the Demonſtration hereof, is the ſame as before.

B
c 14 (e 2
d 4 (3 4

Upon

Upon the same reason is grounded what was added at the end of Division, whereby Pence are reduced to Shillings, Shillings to Pounds, Feet to Paces, Paces to Miles, &c. As if I would reduce 12000 Pence to Shillings; I divide 12000 by 12, and it produces 1000 Shillings, which divided by 20, produces 500 Shillings.

PROBLEM 5.

To reduce a whole Nnmber to a Fraction, whose Denominator is given.

LET the whole Number a, be given to be reduced to a Fraction, whose Denominator may be b.

a 7
b 4
E
28 c
4 d

Multiply the given Denominator b, by the whole Number given a, and under your Product c, write the given Denominator b, thence is made the Fraction E; I say E = a.

Demonstr. by the Definition of Multiplication, Chap. 7. c . . b :: a . . 1; also by *Theorem* 1. c . . b :: E . . 1; wherefore *Euclid* 5. 9. a = E. W. W. D.

8/1 If a Unit be wrote under any whole Number, as 8, it is made a Fraction, or rather something like a Fraction Equivalent to the Integer 8.

Upon the same reason is grounded what was added at the end of Multiplication, whereby Pounds are reduced to Shillings, Shillings to Pence, Miles to Paces, Paces to Feet, &c.

CHAP.

CHAP. IV.

Addition of Fractions.

I.

To add a Fraction to a Fraction.

IF the Fractions (as A, B) have one common Denominator, then add the Numerators 2, 6, together, and write their Sum 8 over the common Denominator 5; and the Fraction C, which arises from thence, is the Sum of the Fractions A and B given,

A $\frac{2}{5}$ $\frac{6}{5}$ B

C $\frac{8}{5}$

If the Fractions have not one common Denominator, as D, E. by *Prob.* 2. Chap. 3. Reduce them to the same Denominator as F, G; the Sum of which H, is equal to D, E, given.

D $\frac{2}{5}$ $\frac{3}{7}$ E

F $\frac{14}{35}$ $\frac{15}{35}$ G

H $\frac{29}{35}$

II.

To add an Integer to a Fraction.

BY *Problem* 5. Chap. 3. Reduce the Integer A, to the Fraction C, having the same Denominator with the given Fraction B; then add, as in Sect. 1.

A 3 $\frac{4}{5}$ B

C $\frac{15}{5}$ $\frac{4}{5}$

D $\frac{19}{5}$

III.

III.

To add many Integers to many Fractions.

ADD the Integers separately into one Summ, and add the Fractions separately into one Summ, as in Section 1. then add these two Summs together as in Section 2.

Let $3\frac{1}{2}$, $4\frac{2}{6}$, $5\frac{4}{9}$, be given to be added together.

$3\frac{1}{2}$		9 9
$4\frac{2}{6}$	18	3 5
$5\frac{4}{9}$		2 8
12		$\frac{22}{18}$
$\frac{216}{18}$	$\frac{22}{18}$	$\frac{216}{22}$ $\frac{238}{18}$

The Summ of the Integers is 12; then to find the Summ of the Fractions in their least terms; I first get the least common Denominator, by *Eucl.* 38. 7. which is 18, then dividing that by the several Denominators I get the Quotients 9, 3, 2. which multiplied by the several Numerators of the Fractions given, produce Numerators to the Denominator found, making Fractions in their least terms and having a common Denominator, equal to those given: then the Numerators 9, 5, 8. added together make 22, under which writing the Denominator 18, it makes the Fraction $\frac{22}{18}$ the Summ of the Fractions given: so is had the Integer 12 and Fraction $\frac{22}{18}$ to be added together, which is done by Section 2. Thus multiply, 12 by 18. and under its Product write 18, then add the Numerators together, and under their Summ write the Common Denominator; so the Fraction $\frac{238}{18}$ is the Summ of the whole Number and Fractions given which was required.

The Demonstration of all this, is sufficiently manifest in it self.

CHAP.

CHAP. V.

Subtraction of Fractions.

I.

To Subtract a Fraction from a Fraction.

IF the Fractions have the same Denominator (as I and K) Subtract the lesser Numerator (2) from the greater (3) and under the remainer 1, write the common Denominator. The Fraction L thence made will be the remainder of the lesser Fraction given I, taken from the greater K.

I $\frac{2}{5}$ $\frac{3}{5}$ K

$\frac{1}{5}$ L

If they have several Denominators (as M, N) reduce them to others O, P, having a common Denominator, by Prob. 2. Chap. 3. which Subtract one from the other, as before, and the remainder will be Q.

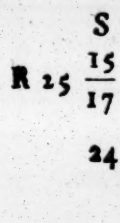
M $\frac{2}{3}$ $\frac{3}{5}$ N

O $\frac{10}{15}$ $\frac{9}{15}$ P

$\frac{1}{15}$ Q

II.

To Subtract a Fraction (less than a Unit, or its whole) from a whole Number.

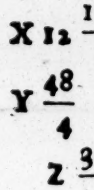


Take a Unit from the Integer R, or a Fraction *b*; *b* equal to a Unit, from which Subtract the given Fraction S, which is done by Subtracting its Numerator *a* from the Numerator *b*, and writing the remainder *c* over the common Denominator *b*, for the Fraction T, arising thence with the whole Number less a Unit, is the remainder sought.

Demonst. From *Corol.* 1. *Theor.* 1. *Chap.* 1. it is manifest that the Fraction *b b* is = to a Unite; wherefore when I Subtract S from *b b*, I Subtract S from a Unite; but it is manifest from Section 1. that S is taken from *b b*; if I Subtract *a* from *b*, therefore S is Subtracted from a Unite, when the Numerator *a* is taken from the Denominator *b*; the rest is clear.

III.

To Subtract a Fraction (greater than a Unit) from a whole Number, or to Subtract a whole Number from such a Fraction.



BY *Prob.* 5. *Chap.* 3. Reduce the Integer X to a Fraction Y, having the same Denominator with the given Fraction V, then Subtract the given Fraction V, from the whole Number so reduced, that is from Y, as in Sect. 1.

After

IV.

To Subtract a whole Number and a Fraction from a whole Number.

BY *Sect.* 2. *Chap.* 4. Collect the whole Number with the Fraction (A) into one Summ (as C) which Subtract from the given whole Number (B) as in the preceeding Section.

A $6\frac{2}{3}$ 16 B

C $\frac{20}{3}$

V.

To Subtract a whole Number from a whole Number and a Fraction.

IF the Integer D, which is to be Subtracted be less than the Integer E to which the Fraction F belongs; subtract the Integers one from the other, and the remainer G with the Fraction given F, shall be the remainer sought.

E F

D 8 13 $\frac{4}{5}$

5 $\frac{4}{5}$

G F

But if the Integer H, which is to be Subtracted, be greater than the Integer I, to which the Fraction K belongs, by *Sect.* 2. *Chap.* 4. Collect the Integer I with its Fraction K into one Summ (as L) and Reduce the Integer H to the Fraction M of the same Denomination with L, by *Prob.* 5. *Chap.* 3. then the Subtraction of M from L is performed as in Sect. 1.

I K

H 12 10 $\frac{9}{2}$

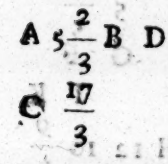
M $\frac{24}{2}$ $\frac{29}{2}$ L

$\frac{5}{2}$ N

Or thus, seeing H is to be Subtracted from I with K; H will be less than I with K; but seeing H is given greater than I; it must of necessity follow that the Fraction exceeds the excess of H above I, and thence is greater than a Unit, wherefore by Prob. 4. Chap. 5. Seek what whole Number there is in the Fraction K, and add it to I, So that O with N may be equal to I with K; now, because H must necessarily be less than O, take H from O, and the remainder P, shall be together with the Fraction N, is the remainder sought.

VI.

To Subtract a whole Number with a Fraction, from a whole Number with a Fraction.



LET the Integer A and its Fraction B, which is to be subtracted, be collected into one Summ C, by Sect. 2. Chap. 4. In like manner get the Summ F, of the other two (D, E,) Then reduce these Summs to the same or a common Denominator, by Prob. 2. Chap. 3. and perform the Subtraction as in Sect. 1.

The Reason of all these Operations is plain from themselves

CHAP.

CHAP. VI.

Multiplication of Fractions.

I.

To Multiply a Fraction by a Fraction.

TO Multiply A by B, Multipy the Numerators *o m* one into another, and their Product is *o m*; also Multiply the Denominators *p*, *n*, one into another, and their product is *p n*, then *o m* shall be the Numerator, and *p n*, the Denominator of the Fraction C, produced by the Multiplication of the Fractions A, B.

A	B
o 2	4 m
p 3	5 n
C	C
8	om
15	pn

II.

To Multiply a Fraction by a whole Number.

TO Multiply D by E, Multiply the whole Number E by the Numerator *k* of the Fraction D, and under the product *n*, write *m*, the Denominator of the given Fraction D, then the Fraction F thence arising shall be the Product sought.

D	E
k 2	5
m 3	1

F

n 10
m 3

Or thus, ſuppoſe a Unit to be wrote under the whole Number E, and the Multiplication will be as in Sect. 1.

III.

To Multiply a whole Number, and a Fraction, by a whole Number.

LET the whole Number with its Fraction be collected into one Summ, by Sect. 2. Chap. 4. then the Multiplication is performed, as in the preceding Section.

Or Multiply the whole Number by the whole Number ſeparately; and then the Fraction by the whole Number.

IV.

To Multiply a whole Number, and a Fraction, by a whole Number and a Fraction.

BY Sect. 2. Chap. 4. get the Summ of one of the whole Numbers and its Fraction, and the Summ of the other whole number, and its Fraction; and multiply theſe two Summs together, as in Sect, 1.

Demonſtration.

THE Operation of the 1 Section is only to be demonſtrated, for the Demonſtration of the reſt will follow from that; it is required therefore to ſhew that B . . C :: 1 . . A, that is, that B multiplyed by A produces C, as is manifeſt from the Defin. of Multiplication, Chap. 7.

$m \times p\,n = p\,n\,m$, also $n \times o\,m = o\,m\,n$, therefore by Theorem 7. Chap. 2. B .. C :: $p\,n\,m$ .. $o\,n\,m$, and by Eucl. 7. 17. $p\,m\,n$ .. $o\,m\,n$ :: p .. o; but by Theo. 1. Chap. 2. 1 : . A : : p . . o, that is B : : C : : 1 .. A W W D.

pnm,	omn,
o m	o m
p n	p n
A, B,	C,

SCHOLIUM.

WHEN the Fractions to be multiplyed together are each less than a Unit, the Product is less than either, which seems strange to them that are not well vers'd in Numbers; but the reason hereof is soon demonstrated from the Definition of Multiplication.

For Example, if $A \times B = C$, *then* 1 . . : A : : B : . C, *therefore if* 1 *be greater than* A, *also* B *shall be greater than* C, *but if* $B \times A = C$; *then again* 1 . . B : : A : : C, *and because* 1 *is greater than* B, *therefore also* A *shall be greater than* C.

$\frac{2}{3}$ A $\frac{3}{4}$ B $\frac{6}{12}$ C

If one of the Fractions to be Multiplied be greater than a Unit, then the Product shall be less than that, but greater than the other.

CHAP. VII.

Division of Fractions.

I.

To Divide a Fraction by a Fraction.

A B C
a 6/8 by m/n 2/4 (o/p 3/2

A B E
a 4/c 9 — by m 3/n 5 — n 5/m 3

D (a n/c m — 20/27

Divide A by B: If the Terms m, n, of the Divisor B, measure the Terms a and c, of the Dividend A; the Numbers o and p, by which they are measured, constitute the Fraction C, and is the Quotient sought.

But if the Terms of the Divisor B, do not measure the Terms of the Dividend A, Invert the Divisor B, and it will be E; then by it so Inverted, multiply the Dividend A, and the Product D shall be the Quotient sought.

Or thus: If the Terms of B, do not measure those of A; multiply the Denominator of the Divisor, by the Numerator of the Dividend, and its Product shall be a new Numerator a n; also the Numerator of the Divisor, multiplied by the Denominator of the Dividend, produces c m, which is a new Denominator, and so the new Fraction D is the Quotient sought.

To

II.

To divide a Fraction by a whole Number, or a whole Number by a Fraction.

$\frac{2}{3}$ by $\frac{6}{1}$ $\frac{1}{6}$ ($\frac{2}{18}$

Suppose a Unit to be placed under the whole Number, and the Division is done, as in Sect. 1.

III.

To divide a whole Number with a Fraction, by a whole Number with a Fraction.

Let each be collected into one Sum, by Sect. 2. Chap. 4. then between the two Sums the Division is performed, as in Sect. 1.

IV.

To divide a large whole Number with a Fraction, by a whole Number.

As for Example: Let 598, with the Fraction A, be given to be divided by 3, you may work this conveniently, thus; divide the Integer 598 by 3, and it quotes 199. if any thing remains, as here 1, add that to the Fraction A, by Sect. 2. Chap. 4. whence you have the Sum B, which divide by 3, as in Sect. 2. and write the Quotient C, to the Integral part of the Quotient, then 199 with C, will be the Quotient sought.

A

3) 598 $\frac{2}{3}$ (199 $\frac{2}{3}$

$\frac{3}{1}$) $\frac{5}{3}$ ($\frac{5}{9}$

B C

Demonstration.

$$\begin{array}{ccc} A & B & C \\ \frac{a}{c} \text{ by } & \frac{m}{n} & \left(\frac{o}{p} \right. \end{array}$$

THE first Operation in *Sect.* 1. only needs Demonstration, for the rest will be manifest from this. In the first place let m, and n, measure a, and c, by o, p; because by Supposition m measures a, by o; therefore by *Euclid* 7. *Axiom* 8. $m \times o = a$: again, because n measureth c, by p, therefore $n \times p = c$; therefore by the Demonstration of the preceeding Chap. $B \times C = A$; therefore by the Definition of Multiplication, $1 \,..\, B :: C \,..\, A$; wherefore by Permutation, $1 \,..\, C :: B \,..\, A$; therefore by the Definition of Division, Chap. 9. Book 1. C is the Quotient of A, Divided by B.

But when the Terms of the Divisor B, do not measure the Terms of the Dividend A, then it seemeth very strange that Division should be performed by Multiplication.

$$F \; \frac{mna}{mnc} \; m\,n$$

$$\begin{array}{ccc} A & B & E \\ \frac{a}{c} \text{ by } & \frac{m}{n} & \frac{n}{m} \end{array}$$

$$D \left(\frac{an}{cm} \right.$$

Wherefore it remaineth to Demonstrate, that if A be multiplied by the Divisor B Inverted, that is by E, the Product D shall be the Quotient of A, Divided by B.

The Terms of the Divisor multiplied one into another, produce mn; that is, $m \times n = mn$, and $mn \times a = m\,n\,a$, and $mn \times c = mnc$, of which the Fraction F is made; by *Euclid* 7. 17. $m\,n\,a \,..\, m\,n\,c :: a \,..\, c$; therefore by Theorem 2. Chap. 2. $F = A$: But the Numerator m of the Divisor B, dividing m n a, gives the Quotient na (for as specious Multiplication is performed only by setting letters together, so division is by taking away of letters) and the Denominator n, of the same Divisor B, dividing mnc gives the Quotient cm; therefore by the first part hereof, the Fraction B divideth the Fraction F, by the Quotient D a Fraction; wherefore since

nce I ſhewed before that F = A, alſo B divideth A by the Quotient D, the ſame that ariſes from the Multiplication of A, by the Diviſor B Inverted; that is by E, which was to be Demonſtrated.

SCHOLIUM.

HEre alſo it ſeems ſtrange to raw Arithmeticians, that in Diviſion of Fractions the Quotient is found for the moſt part greater than the Number which is divided; which always happens, when the Diviſor is leſs than a Unit; but the reaſon hereof is manifeſt from the Definition of Diviſion; A divided by B, gives the Quotient C; therefore by the Definition of Diviſion, 1 . . C : : B . . A; that is by Permutation, 1 . . B : : C . . A, but by Suppoſition 1 is greater than B, therefore alſo C is greater than A.

A B C
5/4 by 2/3 (15/8

CHAP. VIII.

Of Fractions of Fractions.

AS from whole Numbers cut into Parts, ariſe broken Numbers or Fractions, ſo from the parts of whole Numbers ſub-divided in other leſſer parts, are had Fractions of Fractions; as if I were to take A out of B; that is 3 fourths of 16 Twentys of one Pound, they are wrote as in the Examples here ſet; A, B, are 3 fourths of 16 Twentys; C, D, are

A B
3/4 of 16/20
C D
2/5 of 1/6
E F G
2/5 of 1/6 of 3/7

2 fifths

2 fifths of one Sixth; E, F, G, are 2 fifths of one Sixth, of three Sevenths.

For Compendiousness, a Fraction of a Fraction may be called a second Fraction; a Fraction of a Fraction of a Fraction, may be called a third Fraction, *&c.*

Before the Arithmetical Operations of Fractions of Fractions can be Instituted they must first be reduced to simple Fractions.

To Reduce a Second Fraction to a Simple Fraction.

$$B\ \frac{2\ a}{3\ c}\ \text{of}\ D\ \frac{p\ 3}{n\ 4}$$

$$Z\text{—}$$

$$E\ \frac{ap\ 6}{c\ n\ 12}$$

LET the second Fraction B, D, be given to be reduced to a simple Fraction; multiply the Numerators a and p, one into another, which makes a p; and also the Denominators c, n, one into another, which make c n, and the simple Fraction E, made of them, shall be equal to the second Fraction B, D.

Demonstration. Find Z by this Analogy, p . . n : : c . . Z; it is manifest that D is the Whole of the second Fraction B, D, and consequently that B, D . . D : : a . . c; but by Theorem 1. Chap. 2. the simple Fraction D . . 1 : : p . . n; that is by Construction c . . Z; wherefore by equality of reason B, D . . 1 : : a . . Z, again, because by Construction p . . n : : c . . Z; therefore by *Euclid* 7. 19. p Z = cn.

Wherefore seeing p x a = ap, and p x Z = cn, as is shewn before; therefore by *Euclid* 7. 17. a . . Z : : ap . . cn; but before it was shewn that B, D . . 1 : : a . . Z; therefore B, D . . 1 : : ap . . cn; and by Theorem 1. Chap. 3. ap . . cn : : E . . 1; therefore by *Euclid* 5. 11. B, D . . 1 : : E . . 1; therefore by

Euclid

Euclid 5. 9. the second Fraction B, D, and the simple Fraction D, are equal, which was Proposed.

To reduce a third Fraction, or any other to a simple Fraction.

LET the third Fraction C, D, E, be given to be reduced to a simple Fraction.

Reduce the second Fraction D, E, to I a simple Fraction, then C, is the Fraction of the Fraction I; again, reduce the second Fraction C, I, to the simple Fraction G, and it shall be equal to the third Fraction C, D, E given, as is manifest from the Construction it self.

C D E
$\frac{3}{5}\ \frac{a}{b}\ \frac{4}{6}\ \frac{n}{p}$ of $\frac{y}{z}\ \frac{5}{7}$

G $\frac{any}{bpz}$ $\frac{ny}{pz}$ I

The whole Operation is performed after this manner; all the Numerators are multiplied one into another, and also all the Denominators, that is a x n x y = a n y, and b x p x z = b p z, and the Fraction G, thence arising, will be that required.

Questions to Exercise the foregoing Rules of Fractions.

Quest. 1. WHat Number is that, from which if $3\frac{1}{2}$ be subtracted, the remainder will be $4\frac{1}{3}$? *Answer.* $7\frac{5}{6}$.

2. What Number is that which being added to $3\frac{1}{2}$, the Sum will be $7\frac{5}{6}$? *Answer.* $4\frac{1}{3}$.

3. How much is the Fraction $\frac{3}{5}$, greater than the Fraction $\frac{3}{7}$? *Answer.* $\frac{1}{35}$.

4. What

4. What Number is that, which being divided by $4\frac{1}{7}$, quotes $2\frac{1}{8}$? *Answer*, $8\frac{45}{56}$.

5. What part of $2\frac{1}{2}$ is $\frac{3}{4}$? *Answer*, $1\frac{7}{8}$.

6. What Number is that which being multiplied by $3\frac{1}{2}$, produces $12\frac{1}{4}$? *Answer*, $3\frac{1}{2}$.

7. How many Eighteenths, or $\frac{1}{18}$, are there contained in $\frac{5}{9}$? *Answer*, $\frac{10}{18}$.

This is performed by Division, for if $\frac{5}{9}$ be Divided by $\frac{1}{18}$, as in *Sect.* 1. Chap. 3. the Quotient will be $\frac{10}{18}$ required.

8. How many thirds are there contained in $3\frac{1}{2}$? *Answer*, $10\frac{1}{2}$.

9. How many Eighths, or $\frac{1}{8}$, must be added to $\frac{3}{5}$ to make $\frac{8}{9}$? *Answer*, $2\frac{14}{45}$.

To resolve this, first find to what Number $\frac{3}{5}$ must be added to make $\frac{8}{9}$, which is done by subtracting $\frac{3}{5}$ from $\frac{8}{9}$, and you have $1\frac{13}{45}$; then seek how many $\frac{1}{8}$ are contained in $1\frac{13}{45}$, which is found by dividing $1\frac{13}{45}$ by $\frac{1}{8}$, and you have $2\frac{14}{45}$; therefore if $\frac{2}{8}$, and $\frac{14}{45}$ of $\frac{1}{8}$'s be added to $\frac{1}{5}$, the Sum will be $\frac{8}{9}$, as was required.

10. From how many Sevenths, or $\frac{1}{7}$, must $\frac{3}{5}$ be subtracted, so as that $\frac{1}{2}$ may remain? *Answer*, 7 Sevenths, and $\frac{7}{10}$ of $\frac{1}{7}$.

The resolution of this is like the foregoing, using Subtraction instead of Addition.

11. What Number is that, from which if the difference between $\frac{2}{5}$, and $\frac{5}{6}$, be Subtracted, the remainder will be the difference between $\frac{1}{2}$ and $\frac{5}{9}$? *Answer*, $2\frac{2}{45}$.

To resolve this get the difference between $\frac{2}{5}$ and $\frac{5}{6}$, which will be $\frac{13}{30}$; also get the difference between $\frac{1}{2}$ and $\frac{5}{9}$, which will be $\frac{1}{18}$, then add these two differences together, and their Sum is $\frac{2}{45}$, from which Subtract the first difference, to wit, $\frac{13}{30}$, and the remainder shall be the other difference, to wit, $\frac{1}{18}$.

12. By

12. By how many Fifths, or $\frac{1}{5}$, muſt $2\frac{1}{2}$ be multiplied to produce $7\frac{3}{3}$? *Anſwer.* $15\frac{1}{3}$

To reſolve this ſeek a Number which multiplied by $2\frac{1}{2}$, may produce $7\frac{3}{3}$, which will be $3\frac{1}{15}$; then ſeek how many Fifths are contained in $3\frac{1}{15}$, which you will find to be $15\frac{1}{3}$.

13. What Number is that by which if $7\frac{1}{3}$ is divided, the Quotient will be $\frac{5}{8}$ of $17\frac{1}{2}$? *Anſwer* $\frac{8}{35}$.

14. By how many 9's muſt $7\frac{1}{2}$ be divided to quote $2\frac{1}{4}$? *Anſwer.* 30.

CHAP. IX.

Decimal Fractions.

THE Arithmetick of broken Numbers, commonly called Vulgar Fractions, whoſe Theory and Praxis we have already explained, although neceſſary and pleaſant to know, yet what great trouble there is in large Calculations, and many Repetitions, every one knows who are converſant in Numbers; but as the Labour in Diviſion almoſt wholly vaniſhes by *Napiers* Bones; ſo we are freed from the troubleſomneſs of Vulgar Fractions, by the Excellent Invention of *Decimals*; the Praxes of which are performed in an excellent Compendium, as if they were whole Numbers.

What

What Decimal Fractions are, and how to Write them.

DEcimal Fractions or Numbers, are the Tenth, Hundredth, Thousandth, *&c.* Parts of any Whole, that is are such Fractions whose Denominators proceed in a decuple Progression from Unity, as 1, 10, 100, 1000, *&c.* which are wrote after the Vulgar manner like *a*, which is three tenths, and *b*, which is seven hundredths; but because the Denominators of these Fractions have no other Figures but a Unit and Cyphers, the Calculation will be rendred more expeditious; if the Denominators are not expreſt under the Numerators, but over them, by certain Signs or Indices, as is done in Astronomical Minutes; and thence may be called Primes, Seconds, Thirds, Fourths, *&c.* according as the Index denotes, and according to their places reckoned from the left hand, as you see they are expressed hereunder.

$\frac{3}{10}$ a

$\frac{7}{100}$ b

	Primes,	Seconds,	Thirds,	Fourths,	Fifths.
Index.	I	II	III	IV	V
Value.	10	100	1000	10000	100000 *&c.*
	Tens,	Hundreds,	Thouſ.	Tens Thouſ.	Hun. Thou.

I	II	III	IV	V	VI
3	4	6	2	9	4
a	b	c	d	e	f

Therefore the value of the Decimal *a*, *b*, *c*, *d*, *e*, *f*, is this; *a* is three Tenths or Primes; *b* four Hundreds, or Seconds, *c* six Thousandths, or Thirds; *d* two Ten Thousands, or Fourths; *e* nine Hundred Thousands, or Fifths; *f* four Millions, or Sixths: But if to the Figures which are thus marked, or supposed to be

be marked belong others which are not signed as *gh*, then they are to be estimated as whole Numbers, from hence it is manifest that the order of places in any Decimal Fraction proceedeth from the Left hand towards the Right, contrary to the Order of places in Integers, which is from the Right hand towards the Left.

			I	II
8	3	2	1	
g	h	k	l	

When a Decimal Fraction is to be expressed, two things must be wrote, to wit, the Number, and the Index; the Number shews how many Decimal parts must be taken out of the whole, and the Index sheweth what parts.

The end of Decimal Fractions is to perform all Arithmetical Operations, appertaining or belonging to the use of Man, without vulgar Fractions.

CHAP. X.

Requisits needful to the Demonstration of Decimal Arithmetick.

IF we proceed Methodically, the Demonstration of *Decimal Operations*, will be made very easie.

Definitions.

1. THose Figures which have Indices over them, must not be valued according to their places, as whole Numbers, but must be estimated as single Figures, (that is, as if every one stood in the

i	ii	iii
2	3	4
a,	b,	c

the first place) hence if the Decimal Figures *a*, *b*, *c*, are given, although a stands in the Third place, it doth not signify 200 Tenths, but 2 Tenths; Likewise, although 3 standeth in the Second Place, it doth not signify 30 hundredths, but 3 hundredths, *&c.*

2. But if any whole Number be joined to the Decimal Fraction, it must be estimated of the same value, which it would have been estimated at, if the Decimals were wanting; so the Figures *p q*, although they precede the Decimal Figures *a*, *b*, are valued at 48 not 4800.

	i	ii
48	2	9
p q	a	b

3. When the Decimal Fraction is wrote like a vulgar Fraction, the Denominator hath a Unit, and so many Cyphers after it as are denoted by the Index, so the Decimal Fraction *e* wrote after the vulgar way is the Fraction, *n*, *m*, whose Denominator *m*, hath two Cyphers, that is so many as are designed by the Index (11.)

ii	6 n
6	——
e	100 m

4. The Value of a Decimal Index is a Unite with so many Cyphers as is denoted by that Index, as if the Decimal Number K were given, the value of the Index is 1000. this is manifest from Definition 3.

iii
K 3
1000

AXIOME.

IF to any number B, whether it be wholly an Integer, or an Integer and a Decimal Fraction together, there be added Cyphers having decimal indices over them, they shall not change the value of the said Number.

For the Cyphers have no value of themselves, neither being placed before the Integer B, do they raise its place, or increase its value, as is manifest from Definition 2. This Axiom will be illustrated hereafter.

	C		
B	i	ii	iii
23	o	o	o
1	ii	iii	iv
57	o	o	o
B		C	

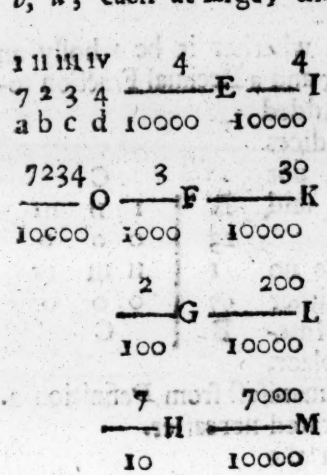
In like manner, when the Decimal Rank A is interrupted, if it be continued by interposing Cyphers, the value of the given Decimal A, will not be changed.

				i	iv
			A.	4.	3 5
1	ii	iii	iv		
4	3	o	o		5

THEOREM 1.

THE Decimal Fraction, a, b, c, d, *is equal to the Vulgar Fraction* O, *whose Numerator is the given Figures* a, b, c, d, (*without the Indices*) *estimated according to their local Value as Integers, and Denominator the value of the greatest Index; that is, a Unit, with so many Cyphers as are denoted by the greatest Index* iv.

Demonſtration. Write the Decimal *Parts*, *d*, *c*, *b*, *a*, each at large; that is, make the Fractions E, F, G, H, whoſe Numerators are the ſaid Figures ſimply taken, but Denominators the values of their Indices. Alſo make other Fractions, I, K, L, M, whoſe Numerators may be the ſame Figures, but eſtimated according to their local value as Integers, and the value of the greateſt Index their common Denominator.



I will now ſhew that the Fractions I, K, L, M, are equal to the Fractions E, F, G, H. Let us compare the two Fractions L and G together: becauſe in whole Numbers the value of places proceeds in a decuple Progreſſion, from the right hand to the left; but in Decimals, the value of Signs or Decimal places proceeds in a decuple Progreſſion from the left hand towards the right; it is manifeſt that the value of the Figure *b*, eſtimated according to its Integral place, (that is, the Numerator 200 of the Fraction L) exceeds the ſimple value of the ſaid Figure *b*, (that is, the Numerator of the Fraction G) as the value of the greateſt Sign (IV) (that is, the Nominator of the Fraction L) exceeds the value of the Sign (II) with which the Figure *b* is marked (that is, the Denominator of the Fraction G) hence 2 . . 100 :: 200 :: 1000; therefore by Theor. 8. Ch. 2. the Fractions L and G are equal; by like reaſon-

4
30
200
7000
7234

reasoning I = E, K = F, M = H. But the Fractions I, K, L, M, constitute the Fraction O, because (10000) the value of the greatest Index (IV) is their common Denominator, and their Numerators being collected together, make 7234, as is manifest in its self; therefore also the Fractions E, F, G, H; that is, the Decimal given *d*, *c*, *b*, *a*, is equal to the Fraction O. W. W. Demonstrated.

THEOREM 2.

IF a whole Number (m, n) *hath annexed to it a Decimal Fraction* (a, b, c;) *the whole Number with the Decimal shall be equal to the Fraction P, whose Numerator is all the Figures given* (m, n, a, b, c) *omitting the Indices where they are, and reckoning the Figures according to the value of their places; and the Denominator* (1000) *the value of the greater Sign* (III.)

Demonstration. By Theor. 1. the Decimal *a*, *b*, *c*, is equal to the Fraction *d*, *e*, and if the whole Number *m*, *n*, be multiplied by *e*, and the product *f* be wrote over *e*, so as to make the Fraction *f*, *g*; the same shall be equal to the Integer *m*, *n*, by Prob. 5. Chap. 3.

	I II III	
	3 2.	5 4 9
	m n	a b c
f	32000	549 d
g	1000	1000 e
	549	
	32000	32549 *b*
	32549	1000 e
		P

But the Fractions *f*, *g*, and *d*, *e*, make the Fraction P. For seeing the Nominator is common to both Fractions, the Numerators *f*, *d*, are only to be added together, whose Sum must of necessity consist of the Figures given, *m*, *n*, *a*, *b*, *c*, as is evident by a small consideration thereof; therefore also the whole Number m, n, with the Decimal Parts *a*, *b*, *c*, make the Fraction P. W. W. Demonstrated.

THEOREM 3.

i	iv		ii v
a 2	3	a	5275
i	ii iii iv	i	ii iii iv v
c 2	0 0 3	c	5 2 0 7 0 0 5
2003 e		5207005 e	
10000 d		100000 d	

WHEN a Decimal Progression a is interrupted, then by Interposing Cyphers, the Interruption of the Progression is taken away, and thence is made c; *and if rejecting the Indices,* e *be wrote over* d, *the value of the greatest Index* (iv; *the Fraction* e, d, *shall be equal to the Decimal Fraction* a.

The Demonstration hereof, is the same as of Theorem 1 and 2.

Corollary.

FROM these Theorems the Axiom before Theo. 1. may be further explained: Take *d*, and to it add three Cyphers, noted with Indices, so as to make *f*; by Theorem 2. d = g h; and *f* = *k m*, by the same Theorem; but by how many Cyphers *k* exceeds *g*, by so many Cyphers *m* must of necessity exceed *h*; therefore by Theorem 1. Chap. 3. Book 2. k . . g : : m . . h, and by permutation, k . . m : : g . . h; therefore by The. 2. Chap. 2. Book 2. the Fraction k m = g h; wherefore seeing k m = f, and g h = d; therefore also, d = f.

i		ii iii iv
d 3 5	f	3 5 0 0 0
g 35	k	35000
h 10	m	10000

This indeed is manifest in its self, and thence the Axiom which I proposed; yet it seemed necessary because of the great use of this Axiom, to write this Declaration thereof.

CHAP.

CHAP. XI.

Addition of Decimals.

LET A, B, C, be given to be added together; if the Progreſſion of the Indices are either interrupted, as in C, or are wanting or break off, as in B, continue the Progreſſion, by interpoſing Cyphers, as in F, or adding Cyphers, as in E; (by which ſupplement the value of C or B are not changed) then write like Indices, under like Indices, as is here done in D, E, F, and add them together as if they were whole Numbers, and over the firſt Figures of the Sum G, write the ſame Indices or Signs, as in the Numbers given to be added, and ſo G, with the Indices, ſhall be the Sum required.

	I II III IV	I II III IV
A	35. 2 4 7 1	D. 35. 2 4 7 1
B	861	I II III IV
		E 861. 0 0 0 0
	II IV	I II III IV
C	7. 4 9	F . . 7. 0 4 0 9
		I II III IV
		G 903. 2 8 8 0

I	K	L
352471	86 10000	70409
10000 p	10000 p	10000 p

N: 903 2880 g / 10000 p

Demonſtration. By the Theorems Demonſtrated in the preceeding Chapter, D, E, F, are equal to the Fractions I, K, L, whoſe Numerators are D, E, F, taken as Integers, and the value of the greateſt Index their common Denominator. Now to get the

the Sum of these Fractions, we need only add the Numerators together, that is, D, E, F, as if they were whole Numbers, and write the common Denominator *p*, under the Sum, as is manifest from Sect. 1. Chap. 4. Therefore, also to get the Sum of the given Decimals, D, E, F, we only need to add them together as Integers, and under their Sum to write *p*, the value of the greatest Index whereby is had the Fraction N; but the Fraction N, by the Theorems of the preceeding Chapter, is equal to the Sum G, for the Numerator thereof hath the same Figures with G, and the Denominator *p*, is the value of the greatest Index (IV) by Hypothesis; therefore G is the Sum of the given Decimals D, E, F, or A, B, C, which was to be Demonstrated.

APPENDIX.

AS the Writing of Decimal Fractions with Indices or Signs over them, doth not want its use and excellency, they best expressing the value of each Figure, yet for common Operation, the writing them is both tedious and perplexing; therefore later Arithmeticians omit the Indices, and reckon the value of the Decimal parts mentally, from their Places or Denominators, setting a Point to the left hand of the Decimal, to distinguish it from a whole Number; which Method we will also joyn to this of using the Indices, in all the Operations of Decimals.

So the Numbers A B C, or D E F, are expressed without Indices, by *d*, *e*, *f*; wherein observe, that like degrees or places are wrote under one another; that is, Primes, under Primes, Seconds under Seconds, and Thirds under Thirds, *&c.* which must always be minded,

d	35 . 2471
e	861 . 0000
f	. . 7 . 0409
g	903 . 2880

ed. And then the Addition is the same as in whole Numbers, only after the Addition is ended, due regard must be had to the placing of the Punctum, to distinguish the whole Numbers from the Decimals; to do which remember this generally, that there must be as many Decimal parts in the Sum, as are in any of the Numbers given to be added; So in this Example, they having each four Decimal Figures, the Point must be placed after the fourth figure in the Sum that is after 2; and so the Sum hath the whole Number 903, and Decimal Fraction 2880.

CHAP. XII.

Subtraction of Decimals.

LET B be given to be Subtracted from A. Write the lesser whole Number under the greater, as usual, and like Decimals one under another, supplying the vacant places in the Indices, both in the beginning, as in A, and in the middle, as in B, with Cyphers signed, as is done in C and D; by which suppletion of the given Decimals A and B, the value is not changed by the Axiom in Chap. 2. then Subtract D from C, as if both were Integral or

	I II		I II III IV V
A	98.42	C	98.42000
	II III V		I II III IV V
B	4594	D	4.05904
			I II III IV V
		Q	94.36096

E $\frac{9842000}{100000}$ F $\frac{405904}{100000}$

T $\frac{9436096\ q}{100000\ p}$

absolute Numbers, and mark the first Figures of the remainder Q, with the same Indices as the other Numbers C or D have, and this shall be the remainder sought.

Demonstration. By the Theorems of Chap. 10. C, D, are equal to the Fractions E, F, whose Numerators are C, D, taken as Integers, and common Denominator p, the value of the greatest Index: Now to Subtract F from E, we need only Subtract the Numerator from the Numerator, that is D taken as an Integer, from C taken as an Integer, and write the common Denominator, that is, the value of the greatest Sign p, under the remainder.

Therefore to Subtract D from C, that is by the Axiom, A from B, we need only to take D as an Integer, from C as an Integer, and under the remainder Q to write p, the value of the greatest Index, whence will be had the Fraction T for the remainder. But the Fraction T, by Theor 2, is equal to Q, for by Hypothesis, the Numerator q, hath the same Figures as Q hath, and the Denominator p is the value of the greatest Sign; wherefore Q is the difference of C and D, or the remainder required. W. W. Demonstrated.

APPENDIX.

d	98. 42000
c	04. 05904
e	94. 36096

THE Numbers above, or any other Numbers having Decimals, may be Subtracted one from the other, without writing the Indices, observing the same Cautions as are given in Addition.

CHAP.

CHAP. XIII.

Multiplication of Decimals.

LET A and B be given to be Multiplied together. The Multiplication of A and B must be performed (without any regard to the Indices) like whole Numbers; (first taking away the interruption of the Decimal Progression, if there be any, by interposing Cyphers, as in the second A) then ad together the greatest Indices of the given Numbers A and B, and their Sum shall give (IV) the greatest Sign by which the first Figure of the Product C must be marked; which Index will also shew how many Figures are to be marked with Indices decreasing orderly.

I III		G	H
7 4 A		704	52
I II III			
A 7 0 4		1000 c	10 p
I			
B 5. 2			36608 k
1 4 0 8			10000 f
3 5 2 0			N
I II III IV			
3 6 6 0 8 C			

Demonstration. By the Theorems of Chap. 10. A and B are equal to the Fractions G and H, whose Numerators are A, B, taken as whole Numbers, and Denominators the values of the greater Signs; but to multiply the Fractions G and H one into another, we need only multiply their Numerators, that is, A and B taken as whole Numbers, one into another, producing a new Numerator *k* (to wit, C without the Indices) and under the same to write the Denominator

nator f, consisting of a Unit, and the Cyphers e, p, taken together; that is the value of both the greatest Signs of A and B together, therefore also to multiply the Decimals A and B together, we only need multiply A and B one into another, as Integers, producing the Numerator k, and under the same write the Nominator f, that is, the value of both the greatest Signs of A and B added together, so getting the Fraction N for the Product: But by Theorem 2. Chap. 10. N = C, for the Numerator k, hath the same Figures that are in C, and the Denominator f, consists of a Unit, and the Cyphers e, p, that is by Hypothesis, of so many Cyphers, as are denoted by both the greatest Signs of A, B together; that is, by Construction, of so many Cyphers as are denoted by the greatest Sign in C; therefore also C is the Product sought by the Multiplication of the given Decimals A, and B, one into the other. W. W. D.

If one of the given Numbers be an Integer, having no Decimals belonging to it; the first Figure of the Product must be signed with the greatest Index of the other Number given, and the rest in order, with Indices decreasing.

APPENDIX.

MUltiplication is better performed for common use without Indices, and then the general directions for Multiplying are these.

Multiply the Numbers given, as if they were whole Numbers, and cut off always from the Product by a Point, so many places towards the right hand, as there are places of Decimal parts in both the Numbers given to be Multiplied; that done, the Figure or Figures (if any happen to be) on the left hand of the said Point, are whole Numbers, and those

thoſe on the right Hand are Decimal Parts.

Let 4.38 be given to be Multiplied by 2.65, that is, a whole Number and a Decimal Fraction, by a whole Number and a Decimal Fraction:

Write the Numbers given to be Multiplied one under another, and multiply them like Integers, ſo getting the Product 116070; then becauſe there are four decimal places in the Multiplicand and Multiplier, cut off four Figures to the right hand from the ſaid Product, ſo it will ſtand thus, 11, 6070; hence the true Product is $11.\frac{6070}{10000}$, or 11.6070.

4.38	
2.65	
2190	
2628	
876	
11.6070	

Let .384 be given to be Multiplied by .269; that is, a Decimal Fraction, by a Decimal Fraction; having got the Product 64896, then becauſe the Multiplicand and Multiplier, have together 6 Decimal parts or places, cut off ſix Figures to the right hand, from the ſaid Product, but here happening to be but five, I muſt add a Cypher to the left hand, and after that place the Point; ſo the Product will ſtand thus, .064896, and are all decimal Parts.

.384	
.269	
3456	
2304	
384	
.064896	

Let .704 be given to be Multiplied by 5.2; that is, a Decimal Fraction, by a whole Number and a Decimal Fraction; having got the Product 36608, becauſe the Multiplicand and Multiplier contain together four places, cut off four Figures to the right hand, from the ſaid Product, ſo it will ſtand thus; 3.6608.

.704	
5.2	
1408	
3520	
3.6608	

The Demonſtration of what is here added, is the ſame as if the Indices were uſed, for the Number of places in the Multiplyer and Multiplicand together are the ſame with the Sum of their greateſt Indices,

25

as is apparent from the Declaration of Decimals in Chap. 9. and from what is added in Addition of Decimals, Chap. 11.

Multiplication of Decimals Contracted.

BEcauſe it oftentimes falls out in the Multiplication of Decimal parts, that there is no need to expreſs all the Figures of the Product, but only ſome of them towards the left hand, thoſe towards the right being but of ſmall value, and in many Operations not any way neceſſary; therefore we will exhibit a Method to Multiply decimal Fractions together ſo, that the Product ſhall have no more parts or places then deſired.

PRAXIS.

WRite the Units place of the Multiplier under that Figure of the Multiplicand, whoſe Index is equal to the Number of Decimal parts deſigned to be left in the Product; then revert the remaining Figures of the Multiplier, and Multiply every Figure of the Multiplier into ſo many Figures of the Multiplicand, as ſtand equal with it towards the left hand, having always reſpect unto what would have been brought, if the next Figure to the right hand in the Multiplicand had been Multiplied, and ſet the firſt Figure of every particular Product under the firſt Multiplying Figure, and the other Figures in order,

Ex.

Example.

LET 5.6839 be given to be Multiplied by 4.329; but so that the Product may have but three Decimal parts.

Having set 4, which is the Units place in the Multiplier, under the third Decimal part of the Multiplicand, because the Product is to have 3 Decimal parts, and reverted the Multiplier, as you see done here: Multiply 5683 by 4, thus; Say 4 by 9 is 36, for which I carry 4 in my Mind; 4 times 3 is 12, and 4 added makes 16, I write down 6 under 4, and proceed as is usual.

5.6839	
9 234	
22736	
1705	
114	
60	
24.615	

Again, to Multiply by 3, say 3 times 3 is 9, for which keeping 1 in my Mind, 3 times 8 is 24, to which 1 being added, it makes 25; then writing 5 under 4, that is under the first multiplying Figure; proceed as is common.

Again, 2 times 8 is 16, for which reserve 2, and 2 times 6 is 12, to which the 2 reserved being added, it makes 14, write down 4 under 5, and proceed.

Lastly, 9 by 6 is 54, for which reserve 6, and 9 in 6 is 54, to which adding 6, reserved, the Sum is 60, the first Figure of which is wrote under 4.

Then the particular Products being added together their Sum is 24615, from which must be cut off three Figures to the right hand for the Decimal parts designed, as you see done in the Scheme; so the Product will be 24.615, consisting of an Integer and but of 3 Decimal Parts, as was required.

Let it be required to multiply 145.684 by 136.38, so as that the Product may have only Integers.

Having

Having set 6, the Units place of the Multiplier under 5, the Units place of the Multiplicand, the first Figure to the right Hand of the Product being to be in the place of Units, revert all the other Figures of the Multiplier which when reverted will stand thus 84631, and then the Work is performed, as in the Example aforegoing, and as is done in the annexed Scheme, producing 19881, without any Decimal Fraction, as was required.

145.684	
84631	
14568	
4370	
874	
58	
11	
19881	

Let 4898 be given to be Multiplied by .3468, and so as that the Product may have two decimal parts, because to the Multiplier there belongs no Integer, therefore set a Cypher in the place of Units; and because in the Multiplicand there are no decimal parts, annex Cyphers thereto, and then the Multiplication will be as in the first Example; see the Scheme, from which all is plain.

4898.00	
8643.0	
146940	
19592	
2939	
391	
1698.62	

Demonst. To Demonstrate this Contraction, set the first Example multiplied at large, and also contracted.

Why the Units place of the Multiplyer 4 is set under that Figure of the Multiplicand 3, whose Index is equal to the Number of places to be in the Product is because Units place having no Index, its Index alone is equal to the greatest Index of the Product.

Also why the first Figure of the Multiplication of 568 by 3, which is 5, must stand under Units, which is the third place of decimal parts, is, because the Indices of 8 and 3, make also three, and the like reason serves for the rest.

Why the Multiplier is reverted, is because the greatest particular Product should stand first, and the rest in order under it; but that it is the same as if not reverted, is manifest, by comparing the two Schemes together.

And why every particular Figure in the Multiplier, multiplies first not that over it, but the next to the right hand, and carrys the tens mentally to be added to the product of the multiplying Figure and that over it, is to make the contraction want as little as may be, of being equal to the Product of the Multiplication at large; which is also manifest, by comparing the two annexed Schemes.

51	1551
113	678
1705	17
22735	6
24.605	6031

111111
5.6839
111 11 1
9 234
22736
1705
114
50
24.605

CHAP. XIV.

Division of Decimals.

```
      i iv v
A. 258 7 9
    i ii iii iv v        i ii
A 2580 0 7 9   B 5 7 3
    iii iv v        i ii iii
  X 4 3 3   (4 5 0 2 C

D 2580079 p   E 573 q
  ---------   ---------
   100000 f    100 g

Z 433         N 4502 h
  ---------   ---------
   100000      1000 k
  ---------   ---------
      i           i ii
h 2 3         m 5 6
    i ii iii iv v      i ii
k 2 3 0 0 0 0   n(4 1 0
```

LET it be required to divide A by B. If the decimal Progression be interrupted, fill the vacancies by interposing Cyphers (as in the second A) and divide A by B, as if they were both whole Numbers; then if the greatest sign of the divisor B, be less than the greatest sign of the dividend A, subtract that from this, and with the remaining Index (iii) mark the first Figure of the Quotient C, and mark the rest in Order, with Indices decreasing.

But if, as in this Example, the greatest Index of the Divisor (*m*) be greater than the greatest Index of the Dividend (*h*) add so many Cyphers to the Dividend (as in *k*) that from their Indices, the Divisors may be Subtracted.

The

The like is done when the Divisor (q) is absolutely greater than the Dividend (p) howsoever the Signs or Indices are.

	1 11
p	8 2 5
	1
q	9 4 2

Demonstr. By Theor. 2. Chap. 10. A = D, and B = E; But to Divide D by E, we need only Divide *p* by *q* (that is A taken as an Integer, by B as an Integer) and *f* by *g*; which last is done by taking the Cyphers in *g*, from those in *f*; that is the greatest Index (11) of the given Number B, from the greatest Index of the given Number A. Therefore also to divide A by B, we only divide A taken as an Integer, by B taken as an Integer; and under its quotient *h*, write a Unit, with those Cyphers which remain when the Cyphers of *g*, are taken from them of *f*; so getting the Fraction N for the Quotient: But by Theor. 2. Chap. 10, N = C. For the Numerator *h* consists of the same Figures as are in C, (for they are both gotten by the Division of the same Numbers) and the Denominator *k* (as I will presently shew) is the value of the greatest Index of C; therefore C is the Quotient arising by the Division of A by B. W. W. D. And that *k* is the value of the greatest Index of the Quotient C, I thus shew; by Construction the greatest Index of C, is that which remained when the greatest sign of B, was taken from the greatest sign of A, but *g* and *f* are the values of the greatest signs of A and B; and so by Defin. 4. they consist of so many Cyphers as are endicated by the greatest signs, or Indices of the said A and B; therefore it is manifest, that if the Cyphers in *g* are taken from the Cyphers in *f*, there will remain *k*, the value of the greatest Index of the Quotient C.

The other case wherein Cyphers are added to the Dividend *h*, to make *k*, we thus demonstrate: By the first part already demonstrated, *m* divideth *k*,

and produces the Quotient *n*, by the Axiom, $k = h$; but *m* dividing *k*, quotes *n*, therefore also *m* dividing *h*, also quotes *n*. W. W: D.

Likewise, from the Construction already delivered it is manifest that when the Divisor is an Integer, the Quotient is noted with the same Indices the Dividend is noted with; also, that when the greatest Indices of the Dividend and Divisor are equal, the Quotient is a whole Number.

2. If when the Division be ended any thing remains, as in Example 1. there remains X, its Figures must be marked with the same Indices, as so many of the first Figures of the Dividend A are marked with.

Demonstr. For A = D, and B = E, but E dividing D, leaveth Z; and by Theor. 1. Chap. 10, Z = X; this is clear, if the first Demonstration be well understood.

3. If the Remainder of the Division be of any Value, annex to it decimal cyphers, and continue *the* Division as in Sect. 1.

| | | |

Demonstra. Because the value of the remainder X is not changed by the annexing of Cyphers, X*t* together shall be equal to X alone; but X*t* divided by B, quotes *r*, therefore also X divided by B, quotes *r*; but A — X divided by B, quotes C, (for A divided by B, leaves X) therefore by dividing all A, by B, the Quotient is C *r*.

The Praxis may be contained in a few words, *viz.* annex certain Cyphers to the remainder of the decimal Division, and continue the Division, and sign the Figures then produced with Indices, orderly increasing from the first Quotient.

Corollary.

After the same manner what remains by dividing a whole Number by a whole Number, may be made to vanish; the Demonstration whereof, is as above.

APPENDIX.

As Multiplication, so Division is better performed for common use without Indices, and the general Directions for Dividing, are these.

Divide the Numbers given, as if they were whole Numbers, and cut off always from the Quotient by a Point, so many places towards the right hand, as being added to the places of decimal Parts in the Divisor, will be equal to the places of decimal Parts in the Dividend; that done, the Figure, or Figures, (if any happen to be) on the left hand of the said Point are whole Numbers, and those on the right, are decimal Parts.

Let it be required to divide 11. 6076 by 4. 38 that is, a whole Number and a decimal Fraction, by a whole Number and a decimal Fraction.

Write the Numbers given to be divided, as in the third way of Division, at the end of Chap. 11. Book 1. and divide as in whole Numbers, so getting the Quotient 265. Then because there are four decimal places in the dividend, and but two in the divisor, cut off so many Figures in the Quotient towards the right hand (to wit two) that being added to the decimal places in the Divisor, the Sum may be equal to those in the Dividend; so the Quotient will stand thus; 2.65.

```
        21
       2849
4.38)11.6070(265
     .8 7680
      2 629   2.65
        21
```

Let it be required to Divide .0125 by .5; that is a decimal Fraction, by a decimal Fraction.

Having divided as in whole Numbers, and got the Quotient 25; then because the Quotient and Divisor together, must contain as many decimal parts as are in the dividend, which here are four; therefore to the two places in the Quotient prefix a Cypher, and after that place the point of separarion, so have you three places of decimal parts in the Quotient, to wit, .025, which with one place in the Divisor, are equal to the places of decimal parts in the dividend.

```
      .
     .2.
.5).0125(25
     105
      2 (.025
```

Let it be required to divide .8564, by .008.

Having

Having divided as in whole Numbers, there remains at the laſt Diviſion 4; and to get its value in the Quotient, add a Cypher (or Cyphers if required) to the Dividend, and continue the Diviſion; the reaſon whereof is demonſtrated in Sect. 3. of this Chap. thento find the value of the Quotient 10705, cut off two Figures to the right hand, which with the three Decimal places in the Diviſor, are together equal to the Decimal places in the dividend, to wit, five including the Cypher annexed; ſo the Quotient will ſtand thus 107.05.

	...4.
.008).	85640(10705
	80600
	5 4 107.05

The Demonſtration, of what is here added, is the ſame, as if the Indices were uſed, for the places of Decimal parts, in the Quotient equal to the difference of the places of Decimal parts, in the Dividend and Diviſor is the ſame with the difference of their greateſt Indices; and that the places of Decimal parts in the Quotient and Diviſor, muſt be equal to thoſe in the Dividend, is manifeſt from the proof of Diviſion; for Multiplying the Diviſor by the Quotient, the Product is equal to the Dividend, hence the Dividend (equal to the Product;) muſt contain ſo many places of Decimal Parts, as are in the Multiplycand (equal to the Diviſor) and Multiplyer (equal to the Quotient) together, therefore having the places of Decimal parts in the Dividend and in the Diviſor, the Decimal parts of the Quotient are conſequently known, being no other then the difference between the places of Decimal parts in the Dividend and Diviſor; which was to be ſhewn.

Division of Decimals contracted.

WHen in Division of Decimal Fractions, the Divisor hath many Figures, and it is required to draw out the Quotient till the remainers of the Division may not be of much value; the operation will be large and tedious, but is excellently contracted by the Methods following, giving as few or as many Decimal parts in the Divisor as is desired.

In this contraction two things are to be considered, to wit, whether the Dividend be absolutely greater than the Divisor, or the Divisor absolutely greater than the Dividend, Let A be given to be divided by B.

B	A	C
2. 31)	5. 2347.	(---.
	14	
	152	
	614	
E 1. 310)	F. 5. 234	(2267
	4620	2667
	462	
	138	
	14	

Having resolv'd how many places of Decimal parts are to be in the Quotient C which let be three, make the places; of Decimal parts in the Dividend and Divisor, each equal thereto by adding Cyphers if they want as in B or cutting of Figures to the right hand if they exceed as in A. so the given dividend and divisor will be E. F. this being done.

If the Dividend A be absolutely greater than the Divisor B, and if the whole Divisor be contained under ten times in the Dividend, as it is in the annexed Scheam; set how often it is contained, in the Quotient which here is 2, and therewith Multiply the Divisor, writing the Product under the Dividend and subtract it from the same; then striking off one Figure to the right hand of the Divisor by placing a point under it as under 0. see how oft the remaining Figures 231 are contained in 614 the remainer

mainer of the Division or new Dividual which is twice, set 2 in the Quotient, and by it multiply the said remaining Figures of the Divisor (adding to the Product the tens (if any) produc'd by that Figure in the Quotient multiplyed by the last Figure cut off, or the next to the right hand) and write the first Figure of its product under the first of the preceeding Product, or of the Dividend, and the other Figures in order; then having Subtracted the Product from the last dividual, proceed and cut off another Figure in the Divisor as 1, by placing a point under it, and with the remaining Figures 23. do as before, so continue lessening the Divisor one Figure to the right hand at every Operation until all have points under them; and so the contracted Division in the annexed Scheam quotes 2267 which by placing a point before the 3 Decimals designed to be in the Quotient, will stand thus 2. 267 or thus 2. $\frac{267}{1000}$.

2. But if the Divisor C be contain'd more than ten times in the Dividend E, place so many Cyphers to the right hand in the Divisor, making E till it cannot be contained 10 times and then proceed in all respects as before, the Quotient in the annex'd Scheam, being to contain 3 decimal parts.

```
                    3
8. 24 )            28
   C )            . 522
                 . 2 270
   E   )        . 36 230 (  54. 263.
8. 2400 )       448. 230 (
               412 000
                 33 960
                  1 648
                    494
                     25
```

3. If the Divisor E be absolutely greater than the

```
23. 0000  E    3
                210
          F 2280
23. 00 )  . 4580 (  1991
  . . . )   2300 (
            2070     . 01991
             207
```

2 D-

Dividend F. and ſo is contained more than tenn times in F. cut off ſo many Figures to the Right hand of the Diviſor till the remaining Figures C are contained leſs then ten times in the Dividend: which done proceed in all reſpects as before.

Obſerve always, that if but one Figure be cut off in the Diviſor, the firſt Figure in the quotient towards the left, is Primes, If two Figures be cut off then place a Cypher for the firſt Figure in the quotient, and the firſt Figure got by Diviſion will be ſeconds: If three Figures be cut off, two Cyphers, and ſo on.

The Praxis of this contraction may be contained in a few words without conſidering how many Fractions are to be in the quotient and is this: having your dividend A, and diviſor B given to be divided contractedly, make the Figures in the dividend and diviſor equal if the firſt Figure to the left of the diviſor be not greater than the firſt Figure to the Left hand of the dividend; but if it be greater then the diviſor, muſt have one Figure leſs than the dividend; this done, proceed in the Diviſion as before directed, ſo getting the quotient 1.9956, to know the value of which is the greateſt difficulty: Therefore to get that, obſerve under what Figure of the dividend, the Unites place of the diviſor would ſtand if wrote under it, for the firſt Figure in the quotient towards the left hand will be of the ſame value, ſo in this Example, the Unites place 8. of the diviſor would ſtand under the Unites place

48.324 | 96.4342
27
.269
4 618
48 110

48.324) 96.434 (1.9956
48 324
43 492
4 349
212
24

place of the Dividend, and therefore the value of 1 is Unites, and all the reſt are Decimals; therefore between 1 and 9, place a point, and the quotient will ſtand thus ; in its proper and true value, 1.9956.

The reaſon of all theſe Rules of contraction is plain from what is Demonſtrated in contracted Multiplycation, *Chap.* 13 For the diviſor multiplyed contractedly by the quotient, muſt of neceſſity produce the dividend, and conſequently the Unites place of the quotient, muſt be under ſuch a place of Decimal parts, in the diviſor, whoſe Index is equal to the greateſt Index of the dividend; and all the other Figures of the ſaid quotient inverted.

Hence in the Firſt Example, becauſe the Units place of the quotient 2, muſt ſtand under the third Decimal place of the diviſor; the Cypher was added to the right hand: and becauſe in the Second Example the Units place 4 in the quotient, muſt ſtand under the third Decimal place of the diviſor, and the place of tens 5.(the quotient being inverted) will ſtand under the fourth Decimal place; therefore two Cyphers are added to the right hand of the diviſor: And in the Third Example, becauſe two Figures were cut off to the right hand of the diviſor the firſt Figures of the quotient reverted muſt ſtand under the ſecond Decimal place of the diviſor; but to make the Product of the Diviſor multiplyed into the quotient, the ſame with the dividend, which hath 4 Decimal places, the Units place of the quotient muſt be wrote under the Fourth Decimal place of the diviſor, which done to fill up the vacancy between 1 and Unity place, you muſt write a Cypher, from which likewiſe the reaſon of the obſervation in *Sect.* 3. is manifeſt.

CHAP. XV.

Reduction of Decimal Fractions.

IF the greateſt Integer of Money, Weight and Meaſure, *&c.* were ſubdivided Decimally, the Doctrine of *Arithmetick* would be taught with much more eaſe and expedition than now it is to eſſay which ſhall add the manner of reducing Fractions, and the Subdiviſions of whole Numbers to Decimal Fractions.

I.

To Reduce a Vulgar Fraction to a Decimal Fraction.

A Single Fraction may be reduced into a decimal of the ſame value, or infinitely near (for all Vulgar Fractions cannot be exactly reduced to Decimals) by *Prob.* 3. *Chap.* 3. *Book* 2. thus. Multiply the Decimals Denominator aſſigned, by the Fractions Numerator, and divide the Product by its Denominator and the Quotient ſhall be the Numerator to the Denominator aſſigned.

Let it be required to reduce $\frac{5}{8}$ to a Decimal, whoſe Denominator is aſſigned to be 1000.

Multiply 1000 by 5, and divide the product 5000 by the Denominator 8 of the Fraction given and the quotient will be 400 which is the Numerator ſought therefore $\frac{400}{1000}$, or $\frac{1}{4}\ \frac{11}{0}\ \frac{111}{0}$ is the Decimal Fraction equal in value to $\frac{5}{8}$.

Let

Let $\frac{7}{240}$ be required to be reduced to a Decimal Fraction, whose Denominator shall be 100000.

Multiply 100000 by 7, and divide the product 700000 by 240, and the quotient will be 2916 and somewhat more; yet less then $\frac{1}{100000}$ of a Unit, which is the Numerator sought, therefore $\frac{2916}{100000}$ or $\frac{II}{2}\frac{III}{9}\frac{IV}{1}\frac{V}{6}$ is the Decimal Fraction almost equal to $\frac{7}{240}$

It will happen in the Reduction of most Vulgar Fractions to Decimals, that the Decimal will want something of being equal to the Vulgar Fraction, and therefore the Denominator of the Decimal must be assigned to be so great that what is wanting in the Numerator may be an inconsiderable Value.

The Demonstration of this, is the same as *Prob.* 3. *Chap.* 3. *Book* 2. which is grounded on *Theo.* 1. *Chap.* 2. *Book* 2. where two Fractions are proved to be equal, if the Numerator of one is to its Denominator as the Numerator of the other is to its Denominator from the inverse of which, to wit, that the Denominator of one Fraction is to its Numerator, as the Denominator of another is to its Numerator, is had the Rule above.

II.

To Reduce the Known Customary parts of Money Weight, &c. to Decimal Fractions.

THE Reduction of the Subdivisions of Money, Weight and Measure, *&c.* to Decimals, differs not at all from that of vulgar Fractions to decimals, for if a whole be divided into parts, and some of them be taken, these are the Numerator of a Fraction whose Denominator is all the Parts; so a Pound *Sterling*, being divided into 20 parts or shillings if 8 of them be taken, they shall be equal to a Fraction, whose Numerator is 8, and Denominator 20.

The

The like is to be understood in every Sub-division of Money, Weight, Measure, &c.

Hence then to Reduce the known Customary parts of Money, &c. to Decimals respect must be had to the Integer Sub-divided; for that being the Denominator and the Parts taken the Numerator, it is reduced to a Decimal, whose Denominator is assigned by the same Rule as above.

Let it be required to Reduce 12 *s.* to the *Decimal* of a Pound, whose Denominator may be 10000 because the Integer a Pound is 20 *s.* by the Rule above 12 Multiplied into 10000, produces 1200000, which being divided by 20, quotes 6000; so shall $\frac{6000}{10000}$ or $\frac{\text{I II III IV}}{6000}$ or $\frac{1}{6}$ (for Cyphers increase not the value of a Decimal) be the Decimal Fraction of a *l.* and is exactly equal to 12 *s.*

Let it be required to reduce 6 *d.* to the Decimal of a Pound, because a pound contains 240 *d.* therefore multiplying the Decimal Denominator 1000 by 6 and dividing the Product by 240 is had the Decimal $\frac{\text{II III}}{25}$ or $\frac{\text{I II II}}{025}$ or $\frac{25}{1000}$ of a pound equal to 6 *d.*

SCHOLIUM.

THe Praxis of the foregoing may be more compendiously exprest thus to Reduce a Fraction, *a*, *b* to Decimals, annex Cyphers to the Numerator *a*, and being so increased divided it by the Denominator *b*; and the quotient *d*, thence arising shall be the Decimal, equal to the Fraction *a*, *b* given; for because by the *Axiome* $a = f$ therefore also *b* dividing, each of them will give equal Quotients; but *b* dividing *a*, quotes *a*, *b*, by *Cor.* 2. *Theor.* 1 *Chap.* 1. and *b* dividing *f*, quotes *d*, by *Hypothesis*, therefore also $a, b = d$.

	I II III
3 *a*	3000 *f*
7 *b*	I II III
	428 *d*

To

To Reduce a Decimal Fraction to another Fraction.

IT is very neceſſary to know, how after any operation is wrought by Decimals, the Product may be eſtimated in another Fraction, or in the known and Cuſtomary Diviſions or parts of whole Numbers as if it were required to know what Fraction having its Denominator 20, the Decimal B is equal to, or which is the ſame thing how many Shillings the ſaid Decimal is valued at, which is obtained after this manner.

Multiply the Decimal 728 given by 20, whoſe Product is 14560, from which take ſo many of the firſt Figures 560, as are indicated by the greateſt Decimal ſign, or as there are places in the given Decimal, and the remainer 14 is the Numerator ſought, therefore the Decimal B makes the C parts of 20 *s.* or one pound, to wit, fourteen twentieths, that is 14 *s.*

	111111
B	728
	20
	14|560
	$\frac{728}{1000}$ A $\frac{14}{20}$

Again to get the Value in pence of the remainer 560, which is a Decimal, having like Indices with that given by *Sect.* of *Multiply. Chap.* Multiply it by 12 cutting of likewiſe ſo many Figures as at firſt, ſo have you remaining 6, which are Pence, and $\frac{720}{1000}$ part of a Peny.

	111111
	560
	12
	6720

After the ſame manner may a Decimal Fraction be reduced to any other Fraction or to the known Subdiviſions of Money, Meaſure, Weight, and time *&c.* the Praxis whereof ſhall expreſs in a few words thus; Multiply the Decimal Fraction given by the Denominator aſſigned, which is the Integer divided into ſuch parts as are deſired, and from

from the right hand cut off ſo many Figures as there are places in the the Decimal given and what remains to the right Hand, ſhall be the Numerator to the Denominator aſſigned or the parts of the Integer required.

Demonſtration. The Fraction A, whoſe Numerator hath the ſame Figures as are in B. and Denominator a Unit with ſo many Cyphers as are deſigned by the greateſt Index of B; is by *Theorem* 1. *Chapter* 10. equal to the Decimal B. But by *Prob.* 3. *Chap.* 3. to reduce the Fraction A to another, whoſe Denominator is 20, the Numerator 728 muſt be multiplied by 20, and the product 14560 divided by the Denominator 1000, which is done by cutting off ſo many of the firſt Figures as there are Cyphers in 2000, that is as are denoted by the greateſt upper Index of the ſaid A; hence therefore the truth of the preſcribed Operation is manifeſt.

The End of the Second Book.

THE

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